Analysis of External Examinations in Mathematics in Montenegro with the Aim of Raising the Quality of the Item Writers' Work

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***Abstract*—This paper presents the analysis of matura and professional exams in mathematics. This is an external exam that students take in Montenegro at the end of high school and vocational school. The paper includes a detailed statistical analysis of some items that are written for the matura and professional exams. Analysis was performed by Classical Test Theory methodology and data processing was taking into account results of all students who took the math on the external exam (about 500 students in gymnasium and about 500 students in vocational school). Items that are listed in this paper represent different levels of competence and domains. Data are presented graphically and show the behavior of items in different groups of students in relation to the total achievement on the test. Based on the results and analysis, clear recommendations are given to the authors how to increase the values of parameters that describe the validity, reliability and discrimination of the items in the test, and also to increase the competence of teachers for creating standardized, valid tests for external verification of students' knowledge.**

***Index Terms*—external exam, analysis of math items, recommendations**

I. Introduction

The Examination Centre of Montenegro is an institution established in 2006.With the aim to prepare and conduct national exams and national assessments in primary and secondary schools [1], [2]. External assessments which are organized in the Centre are conducted in order to verify the level of acquisition of standards in educational programs as to raise the quality of educational system in general [3], [4].

The main activity of the Examination Centre is conducting external matura and vocational exam which is required for acquiring the general i.e. a four-year vocational education, and has selective character when enrolling in faculty [5], [6]. Therefore, the assessment in a standardized and objective way means among other things the preparation of valid, reliable, efficient and acceptable tests.

The following text, on an example of mathematics, describes in which way in Montenegro we come to test items for the purpose of external, i.e. vocational exam.

II. Test Preparation

In accordance with the internal policy of the Centre, math professors (employed at schools or at the University) were informed by public appeals about organizing trainings for creating exam items. The goal of trainings is to create item bank. Training participants were given information about:

- Fields and assessment objectives from Examination syllabus(The Examination syllabus is written according to a certain subject program)
- Types of items
- Difficulty of an item and different levels of cognitive competencies that could be examined by the item(levels of cognitive competencies are defined according to revised Bloom's taxonomy)
- Forms for writing items (how to complete a form which contains subject name, grade, area, assessment objectives, cognitive competencies, type of the item, estimate of item difficulty, estimated time for development, marking scheme, full credit, number of alternatives, ...)
- Manner of delivering items in order to provide privacy and security

The obligation of the participant was to compose a certain number of items after the training. The delivered material is analyzed, a selection is being performed and the chosen items are incorporated in item bank. Criteria for selecting items are:

- Verify the mathematical content defined by assigned assessment objectives from the Examination syllabus which are made based on the standards of the educational program
- Examine the cognitive abilities of a student on different levels in a predetermined relation

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- Along with the formulation of items a detailed solution with precisely defined distribution of marks is enclosed
- Type of the item corresponds the test for the matura, i.e. vocational exam
- The text of the item is clear, comprehensible to the age of a student, and unambiguous.
- Examples are chosen from the bank of redeemed items in order to get a standardized test whose structure meets the criteria indicated in Examination syllabus. It can be concluded that the authors of thus obtained tests are teams of subject specialists.
- After every examination period, in addition to the analysis of results, an analysis of tests and of every item individually is performed. This means that 16 tests and 320 examination items are analyzed. The analysis is performed by a classic test theory. Also, achievements of students who took math exam are being analyzed, "unpublished [7]-[9].
- Montenegro has a small population of students so items cannot be pretested because the preservation of confidentiality could be a problem. That is why a item turns out to be either more difficult or easier than expected, or that students come to the solution in a way that is not provided by the marking scheme, therefore, the gained experience can help authors when creating new items.

III. Examples of Items

Here are some examples of math items with their marking scheme and metrical characteristics. Note that in case of open constructed items we mark the setting of the item, procedure for solving and the correct result. In items of multiple choices, only the final result is marked.

A. Item 1

In a pharmacy $50l$ of syrup is poured into bottles of $\frac{1}{4}l$ and $\frac{1}{8}l$. If a total of 280 bottles were used, how many bottles of $\frac{1}{4}l$, and how many of $\frac{1}{8}l$ were there?

Marking scheme:

x -number of bottles of $\frac{1}{4}\ l$,

y - number of bottles of $\frac{1}{8}\ l$

$x+y=280 \ \wedge \ \frac{1}{4}x+\frac{1}{8}y=50$ .....1 mark

$x=120$ .....1 mark

$y=160$ .....1 mark

Fig. 1 is a graphical representation of this item. The population is divided into 4 homogenous groups based on the total test result so that 1 indicates the group with the lowest score, and 4 indicates the group with the highest score on the test. The item graph displays students' percentage from certain groups who solved the item (open constructed items) or chose a certain alternative (close constructed items) [10].

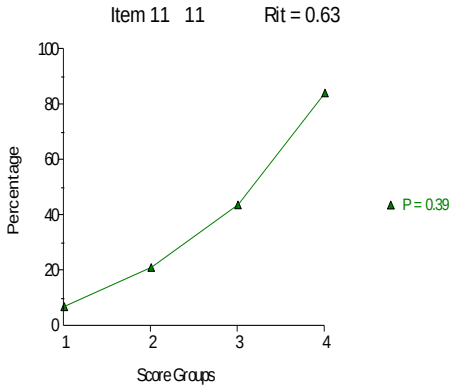

Figure 1. Graphical representation of item 1.

The item was mostly solved by the students from group 4 (around 80%)

The distribution of scores for this item is given on the Table I.

TABLE I. Distribution of Scores for Item 1

0 marks	57%
1 mark	4%
2 marks	3%
3 marks	36%

P -value of this item is 39, while Rit is 63, and Rir 56, which ranks it in the category of items of average difficulty.

The assessment objective of this item refers to implementation of knowledge about solving the system of two linear equations with two variables in solving the problem. The item text does not emphasize the preferred method for solving it, so there were correct answers given by "describing" the solution or by the method of checking which made it difficult to assess the item. We advise the item authors to emphasize which procedure should be used in solving the item, or if they do not insist on a certain procedure, they should give the item in a multiple choice form. Particularly in this case a sum of solution could be asked in order to avoid reducing the item to a mere verification.

B. Item 2

Container in a shape of a cube, whose edge is *a=3dm*, is set so that with one of its edges it touches the horizontal plane, while the steep plane on which the container is set, has a slope 30^o (as shown in Fig. 2). How much water can the container receive in this position?

Marking scheme:

Evaluate short cathetus x:

$$tg30^0=\frac{x}{a} \text{ or } (2x)^2-x^2=3^2 \text{ .....1 mark}$$

$x = \sqrt{3}dm$ 1mark

Volume of prism, i.e. empty part of the container

$V_p = \frac{9\sqrt{3}}{2} dm^3$ 1mark

$V_k = 27dm^3$ 1mark

$V = V_k - V_p = 9\left(3 - \frac{\sqrt{3}}{2}\right) dm^3$ 1mark

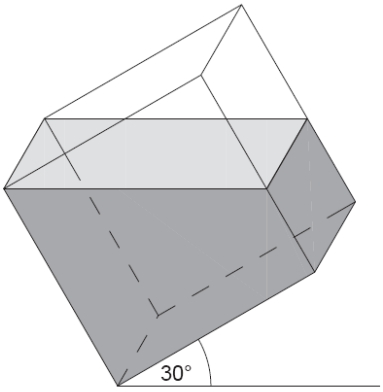

Figure 2. Position of a container.

This item was difficult – its p - value is 29, while Rit is 67, and Rir is 59. Cognitive processes which are used in solving this item are of higher level for the population that is being tested. Distribution of marks is given on the Table II.

TABLE II. DISTRIBUTION OF SCORES FOR ITEM 2

0 marks	43%
1 mark	23%
2 marks	5%
3 marks	7%
4 marks	3%
5 marks	15%

It can be concluded that a relatively small percent of students had 2, 3 or 4 marks which suggests that in similar examples the number of marks could be smaller.

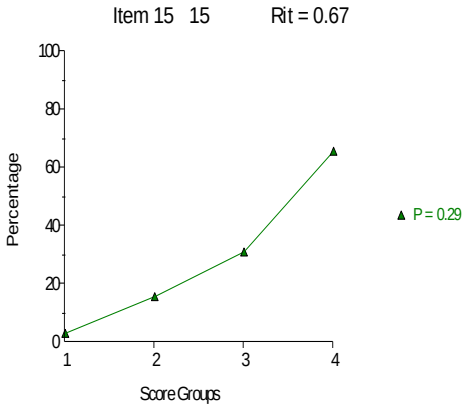

Figure 3. Graphical presentation of item 2.

On Fig. 3 we can see that this request was successfully solved by around 65% candidates of group 4 and less than 5% of candidates of group 1.

C. Item 3

The equation

$$2\log x + \log(x+5) = 2\log(x+2) + \log(x+1)$$

Has two solutions of the same sign;
Has two solutions of different sign;
Has one solution;
Has no solution;

The results show that the item was difficult, its p-value is 28, Rit is 22.

The Table III below shows how the alternatives were selected.

TABLE III. DISTRIBUTION OF SELECTION OF ALTERNATIVES FOR ITEM 3

A	4%
B	11%
C	56%

The analysis of distracters indicates that the first two alternatives are incorrect, yet not evidently incorrect so that no one would chose them. The third alternative attracted a huge number of responds, which tells us that students when solving logarithmic equations neglect the field of logarithmic function definition.

Fig. 4 shows that this alternative was almost equally chosen by the students of all subgroups, both of the students from groups with lowest test results, as well as those with best achievements. Only 28% of students noticed that the result obtained by solving the equation does not belong to the domain. It is recommended that in the future similar items are given in an open constructed form which would allow students who know the procedure of solving logarithmic equations to get a number of marks.

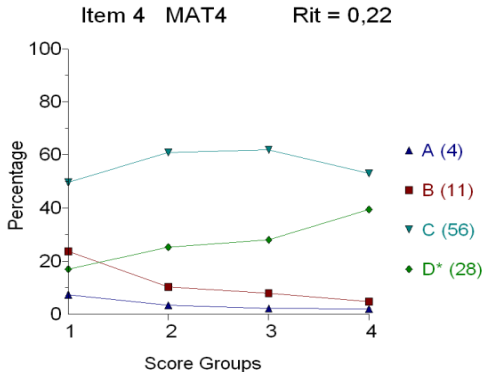

Figure 4. Graphical representation of item 3.

D. Item 4

Evaluate the expression

$$1,23 \cdot 10^3 - (2 + 3 \cdot 8 - 6) + (-5)^0 .$$

Marking scheme:

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$1,23 \cdot 10^3 = 1230$ .....1 mark

$(-5)^0 = 1$ .....1 mark

1211 .....1 mark

The results show that the item is of average difficulty, its p - value is 45, Rit is 67, and Rir is 59.

Distribution by scores is given on the Table IV.

TABLE IV. DISTRIBUTION OF SCORES FOR ITEM 4

0 marks	34%
1 mark	24%
2 marks	13%
3 marks	29%

This item is from the area Numbers. It belongs to the category of items which verify lower cognitive processes. The analysis indicates a very good distribution by marks. The first mark is given for the correct multiplication with the degree of number 10, the second one is given for understanding the rules of grading when the exponent is zero, and the third is given for correct addition and subtraction in the set of integers. Fig. 5 illustrates the performance of this item in different groups of students.

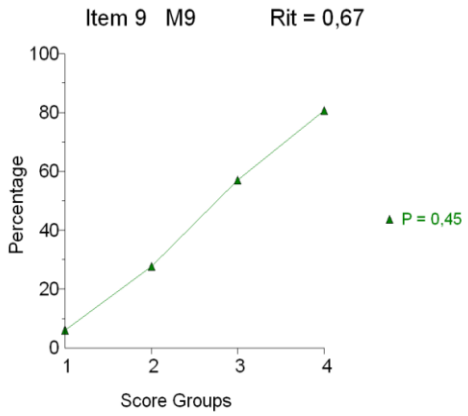

Figure 5. Graphical representation of item 4.

IV. RECOMMENDATIONS FOR TEST AUTHORS

Based on the above analysis of items and tests in general, general and specific recommendations were formed for test authors. Every author, when creating items, must take into account that an item has to examine main educational topics. Students have to solve the item based on their knowledge from mathematics, and not their intelligence or general education. It has to be clear what is required in the item. The requirements are appropriate for the level of the tested population. There cannot be more than one correct answer. Tags and terminology used in items have to be in compliance with those used in the textbook. The item has to be written in a clear and simple style, with concrete instructions and expressions in the question.

Specific recommendations for authors of items are:

- Marking scheme should be done so to avoid fragmentation.
- Avoid items where a large number of ideas lead to their solution. It is very likely that in cases like this the item will not examine the assessment objective which was planned by the test specification.
- Try to formulate items so that the school context is being examined in a creative and original manner.
- Item and marking scheme must be formulated in a way that a student is always rewarded for what he knows. Regarding that, we chose the type of the item; it is open constructed (we want to reward the knowledge of every procedure in solving the item) or closed constructed (we do not insist on procedure, sometimes it is enough that the student knows to solve the problem).
- If the students are not allowed to use the calculator, you should make sure that numerical values, whose aim is not to examine the arithmetic operations in sets of numbers, do not cause a complicated evaluation when solving the problem.
- If possible, authors should join the team of coders in order to get feedback about how students understood items and what were possible ambiguities.

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Zeljko Jacimovic was born in 1966 in Bijelo Polje. He graduated in Chemistry at the Faculty of Science, University of Sarajevo in 1991. He defended his doctoral dissertation in 1999. At the Institute of Chemistry, Faculty of Sciences in Novi Sad. He has been employed at the University of Montenegro since 1992. First as an assistant, and since 2010 as a full professor. He made an outstanding contribution to the reform of pre-university level of education in Montenegro. He was President of the Commission to change the curriculum in elementary school (2003-2005), member of the National Council of curricular and coordinator for fields of natural sciences in the Council (2002-2005), member of the Supervisory Board of Education

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