Performance Measurement–An DEA-AHP Based Approach

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***Abstract*—Performance measurement of various organizations has been addressed by different researchers using various approaches with varied levels and dimensions of consideration. This paper presents a unique approach of combining Data Envelopment Analysis and Analytic Hierarchy Process for evaluating the performance of an organization. The said model overcomes the limitation of DEA model as well as AHP without affecting their unique properties. This technique eliminates the ranking inefficiency of DEA and able to rank all Decision Making Units (DMUs) under consideration. Finally the paper presents an application of the proposed model for measuring the organizational performance with a suitable example of an Indian integrated steel plant.**

***Index Terms*—analytic hierarchy process, data envelopment analysis, performance measurement**

I. Introduction

Due to the rapid emergence of knowledge intensive business, performance measurement in the traditional business organization has become a focal research area [1]. Performance is a multidimensional phenomenon. Primarily, it addresses efficiency, cost, quality, delivery and flexibility aspects relating to the achievement of better performance of an organization. In the current dynamic and vibrant environment of national and global economy, organizational performance is expected to be robust, flexible and competitive enough for its survival, growth and also to have an edge over the competitors.

Performance measurement has been addressed using various approaches from different researchers with varied levels and dimensions of consideration. Different performance measurement tools and techniques have been used including Supply Chain Operations Reference (SCOR) model, Data Envelopment Analysis (DEA), Analytic Hierarchy Process (AHP), Balanced Scorecard (BSC), Analytic Network Process (ANP), and Technique for Ordering Preference by Similarity to Ideal Solution (TOPSIS) etc. Some structural modeling approaches include Interpretive Structural Modeling (ISM), Decision Making Trial and Evaluation Laboratory (DEMATEL) and Structural Equation Modeling (SEM) etc. Benchmarking performance focuses both the intra and inter-organizational levels. In recent past, integrated approaches (SCOR-BSC, BSC-AHP, BSC-ISM-ANP, DEA-AHP model, Fuzzy AHP-Fuzzy TOPSIS, BSC-ANP-DEMATEL, Delphi method-AHP-TOPSIS, Dependence-based interval-valued ER (DIER)-BSC, DEMATEL- ANP- VIKOR) had also been proposed to performance measurement and analysis. Analytic Hierarchy Process (AHP), Data envelopment analysis (DEA) and Analytic network process (ANP) may be considered to be the most popular set of tools for managers engaged in multi criteria decision making.

In this paper, it focuses on performance measurement of a particular organization of longitudinal basis. Much research on performance measurement and improvement shows its degree of importance.

II. Literature Review

DEA and AHP techniques have been extensively used to solve multi criteria decision making problem. There have been limited studies of integrating DEA and AHP model.

Ref. [2] Proposed an integrated DEA-AHP model to evaluate the economic performance of local governments in china and rank different alternatives. In addition, a time-scale comparison of the economic performances of local governments in China was carried out using the Malmquist productivity index (MPI), which indicated that there is a trend of economic growth. Ref. [3] Proposed an integrated fuzzy analytic hierarchy process-data envelopment analysis (FAHP-DEA) for multiple criteria ABC inventory classification using a real case study. This methodology uses the FAHP to determine the weights of criteria, linguistic terms such as Very High, High, Medium, Low and Very Low to assess each item under each criterion, the DEA method to determine the values of the linguistic terms, and the simple additive weighting (SAW) method to aggregate item scores under different criteria into an overall score for each item. Ref. [4] Proposed Fuzzy Analytic Hierarchy Process (FAHP) and Data Envelopment Analysis (DEA) for making bank loan decision on small and medium enterprises in Taiwan along with a practical case study. In this paper, FAHP to choose, the important index in loaning evaluation, establish one complete and efficient loaning decision-making module with its weights and DEA, make effective protection against high ratio of overdue loaning.

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It has a significant influence on banks' saving and loaning business. Ref. [5] Developed a multi criteria decision making aid by using DEA and AHP, which can use efficiently and effectively the internal auditing resources. It focuses on the reliability of the accounting data and evaluates business through financial, operational, and compliance review. It also assesses the risk of asset loss, studies, business processes, and identifies opportunities to improve efficiency and effectiveness. Ref. [6] Presented an integrated DEA and AHP simulation model can be used for selecting optimum alternatives by considering multiple quantitative and qualitative inputs and outputs for a railway system. First, computer simulation is used to model verify and validate the system being studied. Second, the AHP methodology determines the weight of any qualitative criteria (input or outputs). Finally, the DEA model is used to solve the multi objective model to identify the best alternative(s) and also to identify the mechanism to optimize current system. Ref. [7] Used AHP-DEA methodology for assessing the bridge risk. The proposed AHP–DEA methodology uses the AHP to determine the weights of criteria, linguistic terms such as High, Medium, Low, and None to assess bridge risks under each criterion, the data envelopment analysis (DEA) method to determine the values of the linguistic terms, and the simple additive weighting (SAW) method to aggregate bridge risks under different criteria into an overall risk score for each bridge structure. Ref. [8] Applied AHP/DEA methodology to solve a plant layout design problem. The qualitative performance measures were weighted by AHP. DEA was then used to solve the multiple-objective layout problem.

III. DEA-AHP Model for Performance Measurement

The primary objective of this model is to overcome the ranking inefficiency of DEA and eliminates the subjective evaluation of AHP. According to the DEA-AHP method, the judging matrix is formed into using basic DEA models and then AHP is used to rank the DMUs (Decision Making Units). As discussed earlier, it overcomes the ranking inefficiency of DEA and eliminates the subjective evaluation of AHP. This method consists of two steps [9].

Step1 DEA method is used to get the relative efficiency of each pair of DMUs.

Suppose there are *n* decision units and each unit has *m* inputs and *s* outputs

X_{ij} - *i*-th input of *j*-th DMU

Y_{ij} - *i*-th output of *j*-th DMU

Then the DEA method is used to calculate the relative efficiency of each pair of DMUs (without considering the other DMUs). E_{AA} and E_{BA} are the relative efficiency of DMU_A and DMU_B.

$$E_{AA} = max \sum_{r=1}^{s} U_r \; Y_{rA} \qquad (1)$$

$$\begin{cases} \sum_{i=1}^{m} v_i x_{iA} = 1 \\ \sum_{r=1}^{m} u_r y_{rA} \leq 1 \\ \sum_{r=1}^{m} u_r y_{rB} - \sum_{i=1}^{m} v_i x_{iB} \leq 0 \\ u_r \geq 0, r = 1, \dots, s \\ v_i \geq 0, i = 1, \dots., m \end{cases}$$

$$E_{BA} = max \sum_{r=1}^{s} U_r \; Y_{rB} \qquad (2)$$

$$\begin{cases} \sum_{i=1}^{m} v_i x_{iB} = 1 \\ \sum_{r=1}^{m} u_r y_{rB} \leq 1 \\ \sum_{r=1}^{m} u_r y_{rA} - E_{AA} \sum_{i=1}^{m} v_i x_{iA} = 0 \\ u_r \geq 0, r = 1, \dots, s \\ v_i \geq 0, i = 1, \dots., m \end{cases}$$

E_{BB} *and* E_{AB} can be calculated by the same method. Then the relative efficiency ratio of DMU_A and DMU_B

$$a_{AB} = \frac{E_{AA} + E_{AB}}{E_{BB} + E_{BA}} \qquad (3)$$

Generally, there is j row and k column element a_{jk} in the AHP judging matrices:

$$a_{jk} = \frac{E_{jj} + E_{jk}}{E_{kk} + E_{kj}} \qquad (4)$$

And $a_{jj} = 1, a_{kj} = \frac{1}{a_{jk}}$

Step 2 Relative efficiency ratio obtained from step 1 is used to construct Judging Matrix.

Calculate characteristic vector $\omega = (w_1, w_2 ... w_n)^T$ of the judging matrix, $A = [a_{jk}]_{nxn}$ which is obtained from Equation 4, the number j vector is the Wj and it reflects the relative importance level of the number j DMU. This relative importance level is the ranking value of the DMUs, so it is unnecessary to have the consistency test which is mandatory in AHP model.

IV. Application of DEA-AHP Model for Performance Measurement

A. Selection of Input and Output Variables for DEA-AHP Model

The input/output selection in DEA is quite sparse. If inputs and outputs are exogenously specified and if the total number of such variables is large, then DEA efficiency can lose its discriminatory power in the ranking performance of the DMUs relative to each other [10]. In order to avoid model saturation effects, a rule of thumb for selecting an appropriate sample size in DEA is to ensure that it is at least three times larger than the total number of inputs and outputs see Ref. [11]. Many Literatures suggested that the sample size of DMUs should be at least twice the product of the number of inputs and the number of outputs. In this study, eight financial years (2002-2010) of the said organization is considered for performance evaluation based on four variables such as raw material cost; operating and other cost; total volume of sales and profit after tax. The input and output variables used in this study are selected from different literatures which shown in Table I. According to the author raw material cost and operating cost are considered as an input because it represents the amount paid by the organization whereas total volume of sales and profit after tax are treated as outputs since they represent the benefits derived by the organization.

TABLE I. INPUT AND OUTPUT DATA REGARDING THE DMUS

Input/ Output Variables	Issues	References
Raw material cost	Purchasing performance evaluation	Easton *et al.*, 2002
Operational cost	Purchasing performance evaluation Overall performance of supplier	Easton *et al.*, 2002 Garfang, 2006
Total sales volume	Internal Supplier performance evaluation Performance evaluation of distribution center Supply chain performance evaluation Overall performance of supplier	Wong & Wong, 2007 Ross & Droge, 2002 Xu et al., 2009 Garfang, 2006
Profit	Internal Supplier performance evaluation Supply chain performance evaluation	Wong & Wong, 2007 Xu et al., 2009

The output variables:

(1) Total sales-Total saleable steel sales.

(2) Profit after tax-Profit of the organization after paying tax.

The input variables:

(1) Raw material cost-Raw materials consumed by the organization.

(2) Operating and other cost-Stores and spares consumed; fuel oil consumed; repair to building; repair to machine; purchase of power, rent, tax, insurance charge, commission, discount and rebate, wealth tax and other expenses.

B. Collection of Data for DEA-AHP Model

The statistical data used in this paper are adopted from the company's published annual report on a large sized integrated Indian steel plant. Eight financial year data is taken only to explain and validate the said DEA-AHP integrated model. Eight financial years are represented as DMUs and data regarding the DMUs are presented in Table II.

TABLE II. INPUT AND OUTPUT DATA REGARDING THE DMUS

DMUs	INPUTS		OUTPUT	
	Raw material cost (Rs. In Cr.)	Operating & other cost (Rs. In Cr.)	Total sales (Figures in 000' tones)	Profit after tax (Rs. In Cr.)
2002-03	1291	2740	3975	1012
2003-04	1462	3008	4076	1746
2004-05	1715	3687	4074	3474
2005-06	2368	4039	4551	3506
2006-07	3121	4647	4929	4222
2007-08	3430	5069	4858	4687
2008-09	5710	6214	5375	5201
2009-10	5495	6813	6439	5046

SOURCE: COMPANY'S ANNUAL REPORT Ref. [12]

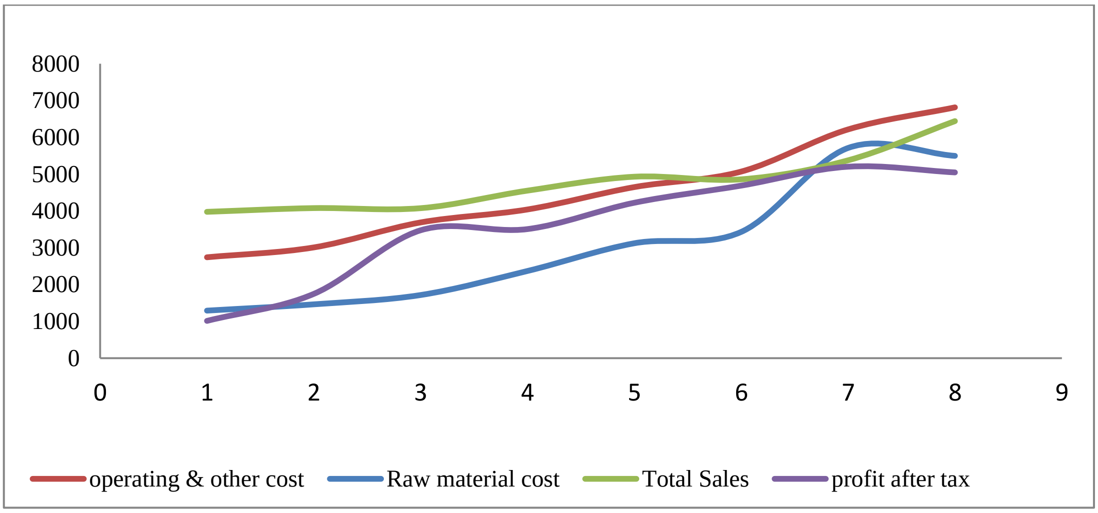

Figure 1. Input and output data

C. Computation of Efficiency and Ranking of DMUS Using DEA

The DEA value (in terms of technical efficiency) of each DMU is calculated by using the software LINGO 8.0 applying the basic CCR principle of DEA method. The ranks of DMUs developed on the basis of DEA value are shown in Table III

TABLE III. RANKS OF DMUS DEVELOPED ON THE BASIS OF DEA VALUE

DMUs	DEA VALUE	DEA RANK
DMU_1	1	1
DMU_2	1	1
DMU_3	1	1
DMU_4	0.9832	4
DMU_5	0.9642	6
DMU_6	0.9813	5
DMU_7	0.8882	7
DMU_8	0.8296	8

Figure 2. DEA efficiency scores of DMUs

D. Computation of Efficiency and Ranking of DMUS Using DEA-AHP Model

The relative efficiency of each pair of DMUs is calculated by using equation (1) and (2). The outcomes of the above computations are used in the equation (4) to construct the Judging Matrix.

TABLE IV. DEA-AHP JUDGING MATRIX

DMUs	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8
DMU_1	1	1	1	1	1	1	1.1438	1.0631
DMU_2	1	1	1	1	1	1	1	1.219
DMU_3	1	1	1	1	1.0371	1.019	1.1258	1.1691
DMU_4	1	1	1	1	1	1	1.0371	1.1720
DMU_5	1	1	0.9642	1	1	1	1.0854	1.1223
DMU_6	1	1	0.9813	1	1	1	1.1048	1.1140
DMU_7	0.8742	1	0.8882	0.9642	0.9213	0.9051	1	1
DMU_8	0.9406	0.8203	0.8553	0.8532	0.8910	0.8976	1	1

Each element of the Characteristic Vector is computed by multiplying together the entities in each row of the Judging Matrix and the nth root of that product is obtained and the value so obtained is then normalized.

Characteristic vector $\omega = (0.1279, 0.1280, 0.1301, 0.1278, 0.1273, 0.1277, 0.1177, 0.1130)^T$

The final rank of DMUs developed using the DEA-AHP model is shown in Table V

TABLE V. RANK OF DMUS DEVELOPED USING DEA-AHP MODEL

DMUs	DEA-AHP VALUE	DEA-AHP RANK
DMU_1	0.1279	3
DMU_2	0.1280	2
DMU_3	0.1301	1
DMU_4	0.1278	4
DMU_5	0.1273	6
DMU_6	0.1277	5
DMU_7	0.1177	7
DMU_8	0.1130	8

It can be seen from Table V that DEA-AHP model differentiates the efficient DMUs as well as inefficient DMUs and develops the relative rank of all DMUs.

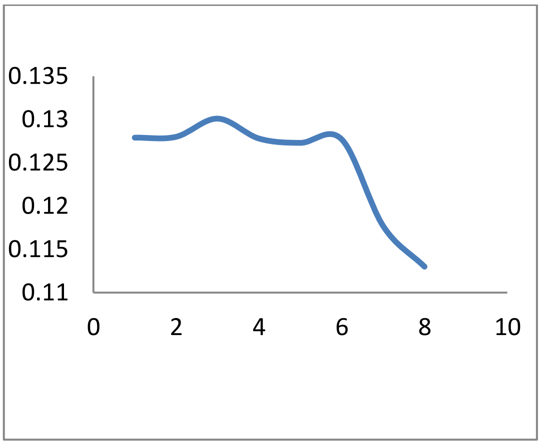

Figure 3. DEA-AHP score of DMUs

V. RESULT AND DISCUSSIONS

Initially, basic CCR based DEA model was used to evaluate the efficiency score for eight financial years of the said organization by considering the input and output variables as performance attributes. The results are depicted in Table III. It is evident that DMUs 1, 2 and 3 are efficient DMUs with a rating of 1.000. The remaining five DMUs are inefficient with ratings of 0.9832, 0.9642, 0.9813, 0.8882 and 0.8296.This result clearly shows that DEA only categorizes the DMUs into efficient (DMUs whose DEA value is equal to one) and inefficient (DMUs whose DEA value is less than one).

Further the DEA-AHP integrated model was used to rank all the DMUs. The AHP judging matrix was constructed by using the relative efficiency score of each pair of DMUs. The characteristic vector was obtained which reflects the relative importance level of each DMU. The vector values were treated as DEA-AHP value and the DMUs were ranked accordingly. This model eliminates the consistency testing, which is a prerequisite in AHP method. This integrated model ranked all the DMUs which have been presented in Table V.

DMU3>DMU2>DMU1>DMU4>DMU6>DMU5>
DMU7> DMU8

The result shows the performance score is high during the financial year 2004-05 and after this year the performance gradually decreases. This is due to the percentage increase in values of output variables is quite rapid than the input variables.

VI. Conclusion

This paper presents a unique approach of DEA-AHP model for evaluating organizational performance. The output of the model highlights its usefulness in the decision making process. This technique is able to rank all DMUs under consideration. The DEA-AHP methodology is simple, easy to use, and applicable to any number of decision alternatives. It is also useful and effective for complex Multi Criteria Decision Making problems with a large number of decision alternatives, where the pairwise comparison is impossible. Further study may be done using more comprehensive data analysis, which may consider several inputs and outputs.

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