Integer Programming Model for Inventory Optimization for a Multi Echelon System

Bassem H. Roushdy
Basic and Applied Science, Arab Academy for Science and Technology, Cairo, Egypt
Email: eng.bassem.roushdy@gmail.com

***Abstract*—The interest in supply chain management and its optimization as complex systems is rapidly growing. In this research, the interaction among different entities in the supply chain is considered where a repeated process of orders and production occur. A supply chain network consisting of multi-echelon manufacturing center and one demand center is considered. A mixed integer programming model is developed to determine production and inventory decisions across different entities supply chain. The objective here is to determine warehouse allocation places in the supply chain to have minimum inventory cost of the system.**

***Index Terms*—supply chains, integer programming, inventory coordination & optimization**

I. Introduction

Inventory optimization remains one of the key challenges in supply chain management. Typically, large amounts of working capital are tied up in today’s supply chains, restricting the opportunities for growth that are essential for a company’s success in competitive markets. However, researches have shown that inventories have a high opportunity to reduce them within supply chain, and hence increasing competiveness factors and reduce production costs.

A supply chain may be defined as an integrated process wherein a number of various business entities like suppliers, manufacturers, distributors, and retailers work together in an effort to: (1) acquire raw materials and component, (2) transform these raw materials into specified final products, and (3) transport these final products to the customer. This process should be under control, there are two types of control; first, the Production Planning and Inventory Control Process; and second, the Distribution and Logistics Process control.

The main task of managing a multi-stage supply chain is to coordinate between different entities in the supply chain. Lean supply chain management provides a means of providing end-to-end synchronization of the supply chain to improve flow and inventory.

Melo *et al.* (2009) classified supply chain network design and optimization into four groups, based on the following: (1) number of stages (single, multiple); (2) number of commodities (single, multiple); (3) number of periods (single, multiple); (4) economic environment (deterministic, stochastic).

Williams (1981, 1983) suggested seven heuristic algorithms for scheduling production and distribution operations in an assembly supply chain network, where each node has only one successor and any number of predecessors. The objective of each heuristic is to determine a minimum production and inventory cost and determine product distribution schedule that satisfies final product demand. The total cost is a sum of average inventory holding and fixed (ordering, delivery, or set-up) costs. He constructed a dynamic programming algorithm to determine the production and distribution batch sizes at each node within a supply chain network.

Cohen and Lee (1989) developed a deterministic, mixed integer, non-linear mathematical programming model using the economic order quantity to develop a global resource deployment policy.

Arntzen *et al.* (1995) presented a Global Supply Chain Model using mixed integer programming model, which can deal with multiple products, facilities, stages (echelons), time periods, and transportation modes. The objective of this model is to minimize a mathematical objective function of: (1) activity days and (2) total fixed and variable cost of manufacturing, material movement, warhorse, and transportation costs.

Voudouris (1996) proposed a mathematical model designed to improve efficiency responsiveness and efficiency of the supply chain. Measurements are based on sum of instantaneous differences between the maximum capacities and the utilizations of two types of inventory resources and activity resources.

Sel çuk *et al.* (1999) suggested a set of decisions used for coordination between different entities in the supply chain. These decisions are applied in manufacturing distribution supply chain. Sawik (2009) stated that decisions concerning ordering, producing, scheduling and distribution should be done simultaneously. Akanle and Zhang (2008) presented a technique form a real manufacturing case study to optimize supply chain configurations to be more flexible with customer demand.

Arshinder *et al.* (2008) & Funaki (2010) proposed a technique for dynamic inventory placement in the supply chain to meet customer demand. The objective is to minimize holding, manufacturing and setup costs.

Almeder *et al.* (2009) showed that combined complex simulation models and abstract optimization models

Manuscript received June 5, 2014; revised August 26, 2014.

doi: 10.12720/joams.4.1.47-52

allow the modeling and solving of more realistic problems, which include dynamics and uncertainty.

Wagner *et al.* (2009) proposed copula approach that acquires a positive supplier default dependencies in the supply chain. They proved that default dependencies are an important factor in risk mitigation strategies and should be put in consideration when selecting suppliers.

Anosike and Zhang (2009) proposed an agent based approach to utilize the resources used for manufacturing under stochastic demand. Bilgen (2010) used a fuzzy mathematical programming approach to determine the optimum allocation of inventory in a manufacturing-distribution supply chain. He applied the model in consumer goods industry.

Benjamin (1989) proposed a simultaneous optimization of production, transport and inventory using a nonlinear programming model. Sousa et al (2008) presented a two-level planning approach for the redesign and optimization of production and distribution of an agrochemicals supply chain network.

Recently, Amin and Zhang (2013) designed a close loop supply chain where planning is carried in two phases. Firstly, a qualitative approach is used to identify possible entities that will integrate the network design phase; secondly an integrated inventory control model that comprises of integrated vendor buyer (IVB) and integrated procurement production (IPP) systems.

The concept of "balancing allocation" in inventories varies from model to model. See, for example, Eppen and Schrage (1981), Federgreun and Zipkin (1984), Jonsson and Silver (1987), Schwarz (1989), Chen and Zheng (1994), Kumar, *et al.* (1995) and Lee (2005). Several authors like Topkis (1969), Ha (1997) and Deshpande et al. (2002) have suggested systems for allocating inventory with successively arriving customers with different priorities for serving classes in a centralized setting.

Customer or demand push is usually defined as a business response in anticipation of customer demand and customer or demand pull as a response resulting from customer demand.

In this paper a pull manufacturing- distribution supply chain network is studied in order to optimize the system under a centralized decision making process, to reach the minimum inventory cost. The amount of inventory in the supply chain can be reduced using a new allocation method; the method is based on the fact that some nodes in the supply chain can carry no inventory. The effect of this decision is measured to examine its effect on lead time, cost and performance. A mixed integer programming model has been developed for solving this problem.

The paper is organized as follows: section two presents the model assumptions and notations used illustrated by drawings; section three focuses on the integer model and; section four, numerical results and analysis are shown; finally section five displays the conclusion and the future work recommended.

II. Model Assumptions and Notations

Notations Used :

D_{nm}: Demand at the most downstream node (n, m) in the supply chain.

Sh: Shortage cost per unit per unit time at the distribution center.

L_{kij}: An integer value that is assigned to arrows at level (k) connecting node (i, j) with node $(j, k+1)$ $L_{kij} = \begin{cases} 0 \\ 1 \end{cases}$.

T_{kij} : The transportation time associated by the arrow L_{kij}.

T_i : The transportation time for receiving all components at node (i) in level one.

M_{ij}: Time required for producing one unit at node (i, j).

p_{ij}: Number of units required from node (i, j) to satisfy one unit of its successor.

D_{ij}: The sum amount of demand ordered at node (i, j) from all the successors.

A_{ij}: An integer value assigned to node(i, j) $A_{ij} = \begin{cases} 0 \\ 1 \end{cases}$.

H_{ij}: Holding cost per unit per unit time for node (i, j).

LT_{ij}: Lead time for node (i, j).

$MLTS_{ij}$: The time needed for node (i, j) to receive all components from suppliers.

A pull supply chain is considered where demand of the marker is a pull of the system for manufacturing. A supply chain is proposed, which consists of (m) vertical echelon levels and (n) horizontal nodes at each level. All nodes in the supply chain are considered to be manufacturing nodes. The last echelon is a manufacturer and demand center in the same time, where demand is generated. Once order is made at the demand center, the demand center begins manufacturing the order if components are available at the warehouse.

Shortage cost occur only at the most upstream node; this cost is related to the time until delivering the order. If the manufacturer carries components required for production, then it will be delivered instantaneously after production. If the manufacturer does not carry inventory then the customer should wait until the manufacturer receive the components and begin production. It is assumed that the waiting time until receiving the orders is the lead time. As waiting time increases, then the probability of losing the customer orders increases. This value is translated to a shortage cost per unit per unit time.

All nodes in the supply chain deliver the required quantity after manufacturing to the downstream nodes. If the node does not carry inventory, then it will wait until it receives all components from its supplier and then production begins.

In this research, an (*s-1, s*) policy is conducted to nodes in the supply chain that carry inventory. The amount of inventory carried t each node is designed to cover the demand during maximum supplier's lead time.

The lead time from a node (i) to another node (j) is determined according to whether it carries inventory or not. If node (i) carries inventory then the lead time equals its manufacturing time plus the transportation time to node (j). If the node does not carry inventory, then the lead time is equal to the transportation time plus the manufacturing time plus the maximum lead time for its suppliers. Nodes do not begin production until they receive all components from their suppliers.

Manufacturing time is determined by the quantity that should be produced, multiplied by the time needed to produce one unit.

The supply chain is considered as network consisting of nodes. Node's location is determined by an (i) value that determines its horizontal level and a (j) value that determines its vertical level. An example is shown in Fig. 1.

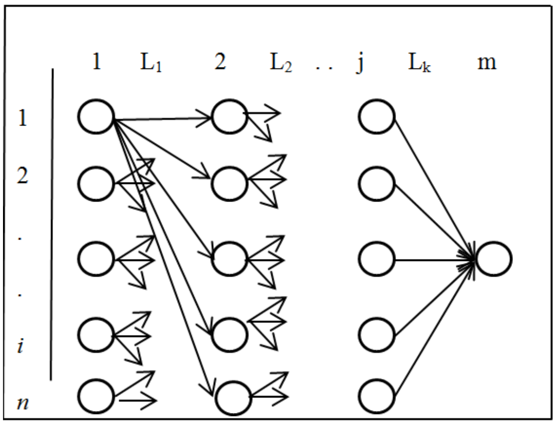


Figure 1. Proposed supply chain structure

To figure axis labels, use words rather than symbols. Do not label axes only with units. Do not label axes with a ratio of quantities and units. Figure labels should be legible, about 9-point type.

Color figures will be appearing only in online publication. All figures will be black and white graphs in print publication.

The location of each node in the supply chain is determined by its value (i,j) in the network. Arc connecting different locations is determined by its vertical location (L) in the supply and the two nodes from the two ends. The symbol L_{111} indicates the arc in level one connecting between node (1, 1) and node (1, 2). Arrow L_{kij} connects node (i,j) and node $(j,k+1)$.

An arrow (L_{kij}) either takes an integer value of {0 or 1}. If a zero is assigned to the arc, then there is no connection between the nodes. If a one value is assigned to the arc, then node (i,j) is a supplier to node $(j,k+1)$. Each arrow holds information concerning the transportation time to the predecessor node. The transportation time (T_{kij}) on arc (L_{kij}) represents the unit time for transporting products from node (i,j) to node $(j,k+1)$. The transportation times to the nodes at the first level $(j=1)$ are given by(T_i).

Each node has an integer value (A_{ij}); whish either takes the value {0 or 1}. The value zero indicates that this node has warehouse and carry inventory. The value 1 indicated that no inventory is carried at this location.

Another value is assigned to each node (M_{ij}); which is the time required to produce one unit of component. And a value (p_{ij}) determines the number of units required by manufacture to produce one unit of the most downstream node demand.

The lead time (LT_{ij}) of node (i,j) is considered to be the time required for manufacturing and receiving components. In case the node carry inventory then it will begin manufacturing directly, else it will wait until components are received to begin production.

Each node has a Maximum Lead Time for its Suppliers $(MLTS_{ij})$. This time determines the maximum time for delivering all components, required for manufacturing, to node (i,j). The $(MLTS_{ij})$ equals to the maximum value of lead times $(LT_{i,j-1})$ of previous level, plus its transportation time to node (i,j).

III. Model Implementation

In this mode, the demand (D_{nm}) occurs only at the most downstream node (demand center); this node is considered to be manufacturer and distributor. The shortage cost is assigned to this location as a function of its lead time. Where (*SH*) is the shortage cost per unit per unit time

$$Shortage\ Cost = SH * D_{nm} * LT_{nm} \tag{1}$$

The lead time (LT_{nm}) for the demand center depends on whether it has inventory or not. If it has inventory, then the lead time equals to its manufacturing time. If it does not have warehouse, and hence no inventory is carried, then its lead time equals the maximum lead time of its predecessors, plus the manufacturing time of the required quantity. Since each node has different transportation time to the demand center (most downstream node) node, so there is no fixed value of lead time to any successor. The $MLTS_{nm}$ determines the maximum lead time of the suppliers to demand center, which is equal to

$$MLTS_{nm} = max.\{LT_{i(m-1)} + T_{(m-1)ij}\} \tag{2}$$

The lead time from the demand center to the customer is the time required for the production of the ordered amount, if the components are available; else it is production time plus the lead time of its suppliers.

$$LT_{nm} = max.\{\, M_{nm}.D_{nm}; A_{nm}.[MLTS_{nm} + M_{nm}.D_{nm}]\} \tag{3}$$

Since each node has different transportation time to its successor nodes, so there is no fixed value of lead time to any successor. The $(MLTS_{ij})$ determines the maximum lead time of the suppliers to node (i,j) including transportation time to this node.

$$MLTS_{ij} = max.\{LT_{ij} + T_{kij}\} \tag{4}$$

Each node (i,j) in the supply chain had a holding cost (H_{ij}) per unit per unit time. The amount of inventory carried at each location is supposed to be the quantity that covers the lead time until orders are received from supplier and then manufactured.

$$LT_{ij} = max.\{\, M_{ij}.D_{ij}.p_{ij};\, A_{ij}.[MLTS_{ij} + M_{ij}.D_{ij}.p_{ij}]\} \quad (5)$$

No holding cost will be incurred at a node that does not carry inventory. While for nodes that carry inventory, the holding cost will be the amount of inventory carried to cover the demand during the $MLTS_{ij}$. The (H_{ij}) is the holding cost for the components required to produce one unit demand (D_{ij}) per unit time.

$$HC_{ij} = \frac{H_{ij}.D_{ij}.MLTS_{ij}}{2} \quad (6)$$

The objective function for this model is to minimize the total system cost. This cost consists of holding cost at each node in the network, plus the shortage cost incurred at the distribution center. The total cost is given by

$$TC = sh.D_{nm}.LT_{nm} + \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{H_{ij}.D_{ij}.MLTS_{ij}}{2}.A_{ij} \quad (7)$$

However, this system is constrained by several factors; first, the demand at each node (i,j) which is determined according to the quantity ordered by its suppliers

$$L_{kij}.p_{i(j+1)}.D_{i(j+1)} = D_{ij}$$
$$\forall_{(i=n\ to\ 1)}, \forall_{(j=m-1\ to\ 1)}, k=j \quad (8)$$

Second, the lead times of nodes in level one of supply chain are constrained by whether the node carry inventory or not. This is shown in the next equation.

Max. $\{T_1 + M_{11} * D_{11} * p_{11}; A_{11} * M_{11} * D_{11} * p_{11}\} = LT_{11}$.
Max. $\{T_2 + M_{21} * D_{21} * p_{21}; A_{21} * M_{21} * D_{21} * p_{21}\} = LT_{21}$.
.
Max. $\{T_n + M_{n1} * D_{n1} * p_{n1}; A_{n1} * M_{n1} * D_{n1} * p_{n1}\} = LT_{n1}$.

$$\text{Max. } \{T_i + M_{i1}.D_{i1}.p_{i1};\ A_{i1}.M_{i1}.D_{i1}.p_{i1}\} = LT_{i1}$$
$$\forall_{(i=n\ to\ 1)} \quad (9)$$

Third factor is the maximum lead time of each node from level $(j = 2\ to\ m)$. The maximum lead time is determined by

$$\text{Max. } \{\ L_{j-1,i,j-1}.(LT_{i(j-1)} + T_{j-1,i,j-1})\} = MLTS_{2j}$$
$$\forall_{(i=1\ to\ n)} \quad (10)$$

The fourth factor is the lead time of the nodes from level two to level (m) in the supply chain, lead time is affected by whether it carries inventory or not. The lead time equation for each node is given by

Max. $\{\ T_{12} + M_{12} * D_{12} * p_{12};\ A_{12}(MLTS_{12} + M_{12} * D_{12} * p_{12} + T_{12})\} = LT_{12}$
Max. $\{\ T_{22} + M_{22} * D_{22} * p_{22};\ A_{22}(MLTS_{22} + M_{22} * D_{22} * p_{22} + T_{22})\} = LT_{22}$
Max. $\{\ T_{n2} + M_{n2} * D_{n2} * p_{n2};\ A_{n2}(MLTS_{n2} + M_{n2} * D_{n2} * p_{n2} + T_{n2})\} = LT_{n2}$
Max. $\{\ M_{ij}.D_{ij}.p_{ij};\ A_{ij}(MLTS_{ij} + M_{ij}.D_{ij}.p_{ij})\} = LT_{ij}\ \forall_{(i=1\ to\ n)}, \forall_{(j=2\ to\ m)}$ (11)

IV. Numerical Results

To solve this optimization problem of this mixed integer supply chain model, a case study has been introduced. A supply chain consisting of four levels echelon inventory system has been considered. The first level consists of raw material procurement centers; each center receives stocks from ample supplier that has deterministic lead time regardless of the quantity ordered. The raw material is manufactured and delivered to the next stage. It is assumed that each node in this stage carries inventory, so once order is received it begins manufacturing directly to supply its successors. If it does not carry inventory, then the center will order quantity needed for manufacturing in synchronize with other nodes in the same level.

The second stage in the supply chain consists of assembly stations; these stations should receive all components from suppliers to be able to begin production. Initially, it is supposed that these nodes will carry inventory and they begin production once orders are received. If the stations don't carry inventory, then they will wait until receiving all components and then begin production. In this case its lead time will increase causing the successor node to carry more stock to cover this period

All stations in the third level directly serve the demand center. The demand center is both manufacture and producer location. The proposed supply chain is shown in Fig. 2.

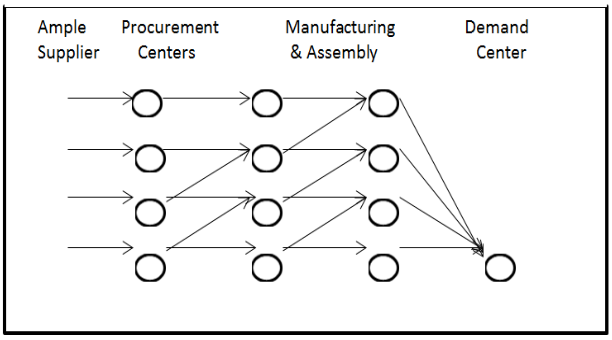


Figure 2. Manufacturing and distribution center case study

The model proposed is programmed using VBA application language, where each node in the supply network has data concerning its predecessors, successors, holding cost per unit, manufacturing time, transportation time to every successor and number of components units required to produce unit demand. A shortage cost is evaluated at the demand center as shown in Table I.

Firstly, all decision variables are assigned a zero value, which means that all nodes in the supply chain will carry inventory in their warehouses. The total inventory and shortage cost for this state is evaluated. A solution procedure based on minimum spanning tree is carried to reach to the optimum state; the procedure begins at the first stage (level l). The decision variable for each node in the supply chain (A_{ij}) is changed to one value, and then the node with highest cost reduction is selected. Starting from the chosen node, the (A_{ij}) value, of the most nearest node with highest cost reduction, is changed to one. This process continues until reaching to the demand center, and then repeated once more.

TABLE I. DATA FOR THE MODEL

	Stage One			Stage Two		
Node	Manufacture Tme/ unit	Holding cost/unit	Componets req./ unit	Manufacture Tme/unit	Holding cost/unit	Componets req./ unit
1	0.05	0.3	1	0.05	2	5
2	0.2	0.2	2	0.01	2	2
3	0.02	0.5	2	0.03	1	2
4	0.03	0.2	3	0.02	1	1
	Stage Three			Stage Four		
	Manufacture Tme/unit	Holding cost/unit	Componets req./ unit	Demand		10
				Holding cost /unit		20
1	0.4	4	2	Shortage cost / unit		50
2	0.4	3.5	2	Componets to produce 1 unit		1
3	0.3	5	3			
4	0.3	4	3			

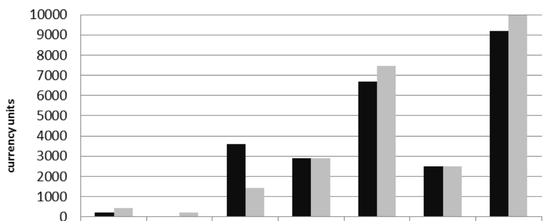
The (A_{ij}) states of each node, after reaching to the optimum state, are shown in Table II. The zero value indicates that this stage carries inventory, while state one indicates that it does not carry inventory. The (A_{ij}) value for each node is shown in Table II.

TABLE II. SUPPLY CHAIN STATE AFTER OPTIMIZATION

i | j	1	2	3	4
1	$A_{11} = 0$	$A_{12} = 1$	$A_{13} = 0$	
2	$A_{21} = 0$	$A_{22} = 1$	$A_{23} = 0$	
3	$A_{31} = 1$	$A_{32} = 1$	$A_{34} = 0$	
4	$A_{41} = 0$	$A_{42} = 1$	$A_{43} = 0$	$A_{44} = 0$

After optimization some nodes will carry extra amount of inventories and will incur higher cost, but in return a drastic cost reduction will occur at other nodes. The resultant supply chain cost is reduced, and hence competitiveness and profits will increase. By the benefits of such reduction, some nodes will be compensated due the cost resulted from holding extra inventory. The performance measure in terms of cost is shown in Table III.

TABLE III. COST REDUCTION AS A PERFORMANCE MEASURE.



	Stage one	Stage Two	Stage Three	Stage Four	Total holding cost	Shortge cost	Total Cost
■ After Optimization	211	0	3582	2900	6693	2500	9193
■ Before Optimization	436	207	1434	2900	7477	2500	9977

V. CONCLUSION AND FUTURE WORK

Supply chain department in companies use one of two methods for their supply chain decision making: the first is the centralized decision making where supply chain decisions are made by the central supply chain department; and the second is the decentralized decision making where supply chain decisions are made at a location separately. Decentralized decision making gives a faster response to changes and customer needs, while the centralized gives lower inventory cost but requires information exchange and high trust.

Decisions in this paper are assumed to be done centrally in the supply chain to take the advantage of cost reduction. Here a supply chain model has been built assuming each manufacturer location as a node that carry inventory. Decisions are done centrally to identify the places of holding inventory and the time of ordering. The model is based on the fact that lead time could be affected by the manufacturing time of each unit and hence the size of the order. The objective here is to allocate inventory in definite places in the supply chain, not at each node. This action was proved that it could increase the lead time to certain extent, but generally cost reduction occurs. The limitation of this model is the rejection of some manufactures to increase its inventory cost.

Considering different production types for each manufacturer can be a good future contribution because of product diversity, product categories, weight and aroma. Expiration dates must also be integrated. Finally, improving the proposed model to handle reactivity will be considered.

REFERENCES

[1] S. H. Amin and G. Zhang, "A three-stage model for closed-loop supply chain configuration under uncertainty," *International Journal of Production Research*, vol. 51, pp. 1405–1425, 2013.

[2] O. Akanle and D. Zhang, "Agent-based model for optimizing supply-chain configurations," *International Journal of Production Economics*, vol. 115, no. 2, pp. 444–460, 2008.

[3] C. Almeder, M. Preusser, and R. F. Hartl, "Simulation and optimization of supply chains: Alternative or complementary approaches," *OR Spectrum*, vol. 31, no. 1, pp. 95–119, 2009.

[4] A. Anosike and D. Zhang, "An agent-based approach for integrating manufacturing operations," *International Journal of Production Economics*, vol. 121, no. 2, pp. 333–352, 2009.

[5] K. A. Arshinder and S. G. Deshmukh, "Supply chain coordination: Perspectives, empirical studies and research directions," *International Journal of Production Economics*, vol. 115, pp. 316–335, 2008.

[6] C. A. Bruce, G. G. Brown, T. P. Harrison, and L. L. Trafton, "Global supply chain management at digital equipment corporation, interfaces," vol. 25, pp. 69-93, 1995.

[7] J. Benjamin, "An analysis of inventory and transportation cost in a constrained network," *Transportation Science*, vol. 23, pp. 177–183, 1989.

[8] B. Bilgen, "Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain network problem," *Expert Systems with Applications*, vol. 37, pp. 4488–4495, 2010.

[9] A. J. Clark and H. Scarf, "Optimal policies for a multi-echelon inventory problem," *Management Science*, vol. 6, pp. 475-490, 1960.

[10] F. Chen, "Decentralized supply chains subject to information delays," *Management Science*, vol. 45, pp. 1076-1090, 1999.

[11] V. Deshpande, M. A. Cohen, and K. Donohue, "A threshold inventory rationing policy for service differentiated demand classes," *Working Paper*, Purdue University, 2003.

[12] G. Eppen and L. Schrage, "Centralized ordering policies in multi-echelon systems with lead times and random demand," in *Multi-Level Production/Inventory Control Systems: Theory and Practice*, L. B. Schwarz, ed. *North-Holland, Amsterdam*, 1981, pp. 26 51-84.

[13] A. Federgruen and P. Zipkin, "Allocation policies and cost approximations for multi location production and inventory problems," *Management Science*, vol. 30, no. 1, pp. 69-84, 1984.
[14] K. Funaki, "Strategic safety stock placement in supply chain design with due-date based demand," *Int. J. Production Economics*, vol. 135, pp. 4–13, 2010.
[15] H. Jonsson and E. Silver, "Analysis of a two-echelon inventory-control system with complete redistribution," *Management Science*, vol. 33, no. 2, pp. 215-227, 1987.
[16] A. Kumar, L. B. Schwarz, and J. E. Ward, "Risk-pooling along a fixed delivery route using a dynamic inventory-allocation policy," *Management Science*, vol. 41, no. 2, pp. 344-362, 1995.
[17] M. T. Melo, S. Nickel, F. Saldanha-daGama, "Facility location and supply chain management—a review," *European Journal of Operational Research*, vol. 196, no. 2, pp. 401–412, 1991.
[18] T. Sawik, "Coordinated supply chain scheduling," *International Journal of Production Economics*, vol. 120, pp. 437–451, 2009.
[19] L. B. Schwarz, "A model for assessing the value of warehouse risk-pooling: Risk pooling over outside-supplier leadtimes," *Management Science*, vol. 35, no. 7, pp. 828-842, 1989.
[20] S. S. Erenguç, N. C. Simpson, and A. J. Vakharia, "Integrated production/distribution planning in supply chains: An invited review," *European Journal of Operational Research*, vol. 115, pp. 219- 236, 1999.
[21] R. Sousa, N. Shah, and L. G. P. Georgiou, "Supply chain design and multilevel planning: An industrial case," *Computers & Chemical Engineering*, vol. 32, pp. 2643–2663, 2008.
[22] D. M. Topkis, "Optimal ordering and rationing policies in a non-stationary dynamic inventory model with n demand classes," *Management Science*, vol. 15, no. 3, pp. 160-176, 1968.
[23] T. V. Vasilios, "Mathematical programming techniques to debottleneck the supply chain of fine chemical industries," *Computers and Chemical Engineering*, vol. 20, pp. S1269-S1274, 1996.
[24] S. M. Wagner, C. Bode, and P. Koziol, "Supplier default dependencies: Empirical evidence from the automotive industry," *European Journal of Operational Research*, vol. 199, no. 1, pp. 150–161, 2009.
[25] F. J. Williams, "Heuristic techniques for simultaneous scheduling of production and distribution in multi-echelon structures: Theory and empirical comparisons," *Management Science*, vol. 27, no. 3, pp. 336-352, 1981.
[26] F. H. Williams, "A hybrid algorithm for simultaneous scheduling of production and distribution in multi-echelon structures," *Management Science*, vol. 29, no. 1, pp. 77-92, 1983.

Bassem Roushdy is working as a lecturer in the Department of Industrial Engineering in the Arab Academy for Science and Technology, Cairo, Egypt. He is awarded his Ph.D. form the Department of Mechanical Engineering in Ain Shams University in 2012. Since that time he is working in the field of supply chain optimization and jobs scheduling. Dr. Bassem has experience of executive leadership in manufacturing, supply chain management, quality control, continuous improvement and decision making.