Robust Design of Mixing Continuous-Continuous and Continuous-Digital Type Dynamic Systems

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***Abstract*—The objective of robust design is to minimize the total quality loss in products or processes. This paper resolves the relationship between the quality loss function and the SN ratio. According to definitions of the quality loss function and SN ratio, the average SN ratio for a factor level can be transformed into a PQL value. The PQL model proposed by this study simultaneously optimizes the continuous-continuous and continuous-digital dynamic systems by minimizing the total quality loss. Using the proposed method and steps for optimization, the method addresses such problem to improve the product quality.**

***Index Terms*—robust design, continuous-continuous and continuous-digital dynamic systems, quality loss function, SN ratio, PQL**

I. Introduction

There are several methods to reduce the effect of noise factors and unit-to-unit variation in order to improve the performance of products. Among these methods, the robust design proposed by Taguchi [1] is a particularly cost-effective approach for helping manufacturers to design their products, leading to higher customer satisfaction and operational performance. The robust design problem is classified as a static system if the desired output of the system has a fixed target and as a dynamic system if the desired output of the system is a function of signal factor settings. The goal in optimization for the dynamic system is to minimize error variance while keeping the response on target.

Several researchers have studied the robust design of dynamic systems (see Chang [2], Chen [3], Lesperance and Park [4], Li [5], Lunani, Nair and Wasserman [6], McCaskey and Tsui [7], Miller and Wu[8], Nair, Taam and Ye [9], Tsui [10], Wasserman [11], Wu and Yeh [12], Wu [13], [14], Wu, Wang and Fan[15]). However, few studies have sought to simultaneously optimize robust design for the continuous-continuous and continuous-digital dynamic systems.

This paper proposes a PQL (proportion of quality loss for a factor level) index based on the factor effects of SN ratios. The PQL converts the multiple quality characteristics into a problem with a single characteristic by minimizing the total PQL value to obtain the optimal parameter conditions.

II. Continuous-Continuous and Digital-Continuous Dynamic Systems

In a dynamic problem, if a product or process to be optimized has a signal input that directly decides the output, the optimization involves determining the best control factor levels so that the “input /output” ratio is closest to the desired relationship.

The continuous-continuous dynamic problem means that both the input setting and the output quality characteristic take positive or negative continuous values. The continuous-digital dynamic problem means that the input setting is continuous and the output quality characteristic is discrete. For example, the input temperature setting is continuous and the output, ON or OFF state of the heating unit, is discrete. Such problems can be divided into two separate problems: one for the ON function and the other for the OFF function.

III. Robust Design Model

The quality loss has two components in the continuous-continuous dynamic system. One is due to the slope being other than one and the other is due to the deviation from linearity. Hence, Taguchi [1] defined the SN ratio for the continuous-continuous dynamic characteristic as

$$\eta = -10\cdot\log\left(\overline{L_a}(\mathbf{y})\right) = -10\cdot\log\left(K\cdot\sigma_e^2/\beta^2\right)\ \text{(dB)} \quad (1)$$

where $\overline{L_a}(\mathbf{y})$ is the quality loss after adjustment, β is the slope, σ_e^2 is the error variance and K is loss coefficient for continuous-continuous dynamic characteristic.

Wu [13] revised the SN ratio of digital-digital dynamic characteristic after leveling operation proposed by Taguchi as

$$\eta_{af} = -10\cdot\log\left(K_1\frac{p'}{1-p'} + K_2\frac{q'}{1-q'}\right)$$
$$= -10\cdot\log\left(\left(K_1+K_2\right)\frac{p_I}{\left(1-p_I\right)}\right)\text{(dB)} \quad (2)$$

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where K_1 and K_2 are the loss coefficients of processing one accurate output.

The effect of a factor is defined as the deviation it causes from the overall mean. Let $\bar{\eta}_{i,j}$ be the average value of the SN ratio for the j-th level of factor X_i, let $\bar{\eta}_{i,0}$ be the average value of the SN ratio for the starting conditions of X_i and let $\bar{T}$ be the total average value for the SN ratio. The effect of the j-th level of X_i is defined as

$$\bar{\eta}_{i,j} - \bar{T} = \delta_{i,j} \tag{3}$$

Suppose that there are q control factors, $\mathbf{X} = (X_1, X_2, \ldots, X_q)$, for a single quality characteristic. The effects of SN ratios for some parameter conditions and starting conditions are ($\delta_{1,j_1}, \delta_{2,j_2}, \ldots, \delta_{q,j_q}$) and ($\delta_{1,0}, \delta_{2,0}, \ldots, \delta_{q,0}$), respectively. Then, the estimated values for the SN ratio are presented as follows:

$$\eta = \bar{T} + \delta_{1,j_1} + \delta_{2,j_2} + \cdots + \delta_{q,j_q} \tag{4}$$

$$\eta_0 = \bar{T} + \delta_{1,0} + \delta_{2,0} + \cdots + \delta_{q,0} \tag{5}$$

The relationship between ($\delta_{1,j_1}, \delta_{2,j_2}, \ldots, \delta_{q,j_q}$) and ($\delta_{1,0}, \delta_{2,0}, \ldots, \delta_{q,0}$) is expressed as

$$\delta_{i,j} - \delta_{j,0} = \left(\bar{\eta}_{i,j} - \bar{T}\right) - \left(\bar{\eta}_{i,0} - \bar{T}\right) = \bar{\eta}_{i,j} - \bar{\eta}_{i,0} \tag{6}$$

Let L be the quality loss for some parameter conditions and let L_0 be the quality loss for the starting conditions. The ratio of L to L_0 (proportion of quality loss, PQL) is defined as

$$\mathrm{PQL}_{\mathbf{X}} = \frac{L}{L_0} = \frac{K \cdot 10^{-\eta/10}}{K \cdot 10^{-\eta_0/10}} = 10^{-(\eta-\eta_0)/10} \tag{7}$$

Consider the effect of each factor in $\mathrm{PQL}_{\mathbf{X}}$ with a corresponding starting $\mathrm{PQL}_{\eta_{i,j_i}}$. Equation (7) can be rewritten as

$$\mathrm{PQL}_{\mathbf{X}} = \prod_{i=1}^{q} 10^{-\left(\delta_{i,j_i} - \delta_{i,0}\right)/10} = \prod_{i=1}^{q} 10^{-\left(\bar{\eta}_{i,j} - \bar{\eta}_{i,0}\right)/10}$$

$$= \prod_{i=1}^{q} \mathrm{PQL}_{\eta_{i,j_i}} \tag{8}$$

Suppose that a product or process has p quality characteristics, $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_p)$. Let $\bar{L}(\mathbf{Y}) = (L_{1,0}, L_{2,0}, \ldots, L_{p,0})$ be the average quality loss and let $\mu_{\boldsymbol{\eta}} = (\eta_{1,0}, \eta_{2,0}, \ldots, \eta_{p,0})$ be the predicted SN ratio, for the starting conditions. Therefore, the parameter conditions, $\mathbf{X}_{opt} = (X_{1_{opt}}, X_{2_{opt}}, \ldots, X_{q_{opt}})$, represent the optimal robust design for multiple quality characteristics, by minimizing total quality loss. That is, is,

$$Min: \bar{L}(\mathbf{Y}) = \sum_{i=1}^{p} L_{0_i} \cdot \mathrm{PQL}_{\mathbf{X}_{opt}} \tag{9}$$

To solve the problem for multiple quality characteristics, an optimization procedure is proposed as follows.

Step1: Compute the SN ratio for each quality characteristic and then calculate the main effect of the factors for each quality characteristic.

Step2: Estimate the average SN ratio (η_0) for the starting conditions, for each quality characteristic.

Step3: Transform the SN ratios into a PQL, for each quality characteristic.

Step4: Estimate the quality loss of the starting conditions for each quality characteristic and then program a search module, using EXCEL VBA, to obtain the optimal parameter conditions.

IV. IMPLEMENTATION

A temperature controller can be divided into three main modules: (1) temperature sensor, (2) temperature control circuit, and (3) heating (or cooling) element. The function of the temperature sensor is to measure the temperature accurately and pass that information to the temperature control circuit. The temperature control circuit provides a way of setting the target temperature. It also compares the observed temperature with the target temperature and makes a decision about turning the heater ON or OFF. The threshold resistance, $R_{\text{T-ON}}$, at which the heater turns ON and the threshold resistance, $R_{\text{T-OFF}}$, at which the heater turn OFF were selected as the quality characteristics. The control factors of experiments are listed in Table I. There are six noise factors (R_1, R_2, R_4, R_5, E_0 and Ez); each noise factor has 3 alternate levels(multiply by mean for noise factors). The nominal values of the circuit parameters under the starting settings, their tolerance (three-standard-deviation limits), and the three levels of testing are listed in Table II.

The computed error rates of ON-OFF function and standardized error rate p_I are listed in Table III and the computed SN ratios (η) for all quality characteristics are listed in Table IV.

TABLE I. CONTROL FACTORS AND THEIR LEVELS OF TEMPERATURE CONTROL CIRCUIT

Factors	Level 1	Level 2	Level 3
A. Resistance R_1 (kΩ)	1.95	<u>3.9</u>	7.8
B. Resistance R_2 (kΩ)	3.75	<u>7.5</u>	15.0
C. Resistance R_4 (kΩ)	1.65	<u>3.3</u>	6.6
D. Resistance R_5 (kΩ)	180.0	<u>360.0</u>	720.0
E. Nominal Voltage Ez (V)	5.0	<u>5.3</u>	5.6

*Starting levels are identified by underscore

The main effects of factors of computed SN ratios (η) for all quality characteristics are listed in Table V~Table VII.

Using equation (6), the factor effects of SN ratios can be transformed to a PQL for each quality characteristic and these are shown in Table VIII~Table X.

TABLE II. NOISE, SIGNAL FACTORS AND THEIR LEVELS OF TEMPERATURE CONTROL CIRCUIT

Factor	Mean*	Levels		
		1	2	3
A: R_1 (kΩ)	3.9	1.0408	1.0	0.9592
B: R_2 (kΩ)	7.5	1.0408	1.0	0.9592
C: R_4 (kΩ)	3.3	1.0408	1.0	0.9592
D: R_5 (kΩ)	360.0	1.0408	1.0	0.9592
E: Ez (V)	5.3	1.0408	1.0	0.9592
F: Supply Voltage E_0 (V)	10	1.0408	1.0	0.9592
G: Resistance R_3 (kΩ) (Signal factor)		0.5	1.0	2.0
H: Temperature (Signal factor)		20 ℃	30 ℃	

* a. Mean values listed here correspond to the nominal values for the starting setting

TABLE III. THE ERROR RATES OF ON-OFF FUNCTION

Expt. No.	Factor assignment					20℃	30℃	Standardized error rate p_I
	A	B	C	D	E	p	q	
1	1	1	1	1	1	0.00708	0.00489	0.00588
2	2	2	2	2	2	0.02010	0.01281	0.01605
3	3	3	3	3	3	0.00834	0.00813	0.00823
4	1	2	2	3	3	0.04111	0.01675	0.02631
5	2	3	3	1	1	0.01226	0.00270	0.00576
6	3	1	1	2	2	0.00348	0.00098	0.00185
7	2	1	3	2	3	0.01799	0.00307	0.00745
8	3	2	1	3	1	0.00330	0.00090	0.00172
9	1	3	2	1	2	0.04438	0.00657	0.01723
10	3	3	2	2	1	0.05150	0.00593	0.01768
11	1	1	3	3	2	0.03797	0.00425	0.01281
12	2	2	1	1	3	0.05979	0.00550	0.01841
13	2	3	1	3	2	0.04620	0.00362	0.01310
14	3	1	2	1	3	0.04872	0.00342	0.01309
15	1	2	3	2	1	0.04206	0.00319	0.01172
16	3	2	3	1	2	0.03417	0.00347	0.01098
17	1	3	1	2	3	0.00551	0.00365	0.00448
18	2	1	2	3	1	0.00399	0.00344	0.00370

TABLE IV. THE SN RATIOS(dB) FOR $R_{T\text{-}ON}$, $R_{T\text{-}ON}$ AND ON-OFF FUNCTION

Expt. No.	Factor assignment					$R_{T\text{-}ON}$	$R_{T\text{-}ON}$	ON-OFF function
	A	B	C	D	E			
1	1	1	1	1	1	16.7642	26.8928	19.2688
2	2	2	2	2	2	20.3043	26.8928	14.8644
3	3	3	3	3	3	23.2471	26.8928	17.7985
4	1	2	2	3	3	20.3083	26.8777	12.6723
5	2	3	3	1	1	22.9438	26.9552	19.3598
6	3	1	1	2	2	16.7813	26.8777	24.3173
7	2	1	3	2	3	23.277	27.0096	18.2350
8	3	2	1	3	1	16.7733	26.5124	24.6296
9	1	3	2	1	2	20.162	26.5356	14.5525
10	3	3	2	2	1	20.2696	26.5288	14.4385
11	1	1	3	3	2	23.2608	26.9907	15.8586
12	2	2	1	1	3	16.7562	26.5288	14.2593
13	2	3	1	3	2	16.7617	26.2083	15.7610
14	3	1	2	1	3	20.3482	27.1335	15.7625
15	1	2	3	2	1	23.2176	27.1335	16.2496
16	3	2	3	1	2	23.2355	27.1816	16.5377
17	1	3	1	2	3	16.7472	26.2088	20.4543
18	2	1	2	3	1	20.3197	27.0930	21.2896

TABLE V. SUMMARY OF FACTOR EFFECTS FOR $R_{T\text{-}ON}$ (SN RATIO)

Level \ Factor	A	B	C	D	E
Level 1	20.0767	**20.1252**	16.7640	20.0350	20.0480
Level 2	20.0605	20.0992	20.2853	20.0995	20.0843
Level 3	**20.1092**	20.0219	**23.1970**	**20.1118**	**20.1140**

* Starting levels are identified by underscore
Optimal parameter levels for each characteristic are identified by boldface type

TABLE VI. SUMMARY OF FACTOR EFFECTS FOR $R_{T\text{-}OFF}$ (SN RATIO)

Level \ Factor	A	B	C	D	E
Level 1	26.7732	**26.9996**	26.5382	**26.8713**	**26.8526**
Level 2	26.7813	26.8545	26.8436	26.7752	26.7811
Level 3	**26.8545**	26.5549	**27.0272**	26.7625	26.7752

TABLE VII. SUMMARY OF FACTOR EFFECTS FOR ON-OFF FUNCTION (SN RATIO)

Level \ Factor	A	B	C	D	E
Level 1	16.5093	**19.1220**	**19.7817**	16.6234	**19.2060**
Level 2	17.2948	16.5355	15.5966	**18.0932**	16.9819
Level 3	**18.9141**	17.0607	17.3398	18.0016	16.5303

According to equation (7), the PQL values for the optimal parameter conditions for each quality characteristic are described as follows:

TABLE VIII. SUMMARY OF FACTOR EFFECTS FOR $R_{T\text{-}ON}$ (PQL)

Level \ Factor	A	B	C	D	E
Level 1	0.9963	**0.9940**	2.2497	1.0150	1.0084
Level 2	1.0000	1.0000	1.0000	1.0000	1.0000
Level 3	**0.9888**	1.0180	**0.5115**	**0.9972**	**0.9932**

TABLE IX. SUMMARY OF FACTOR EFFECTS FOR $R_{T\text{-}OFF}$ (PQL)

Level \ Factor	A	B	C	D	E
Level 1	1.0019	**0.9671**	1.0729	**0.9781**	**0.9837**
Level 2	1.0000	1.0000	1.0000	1.0000	1.0000
Level 3	**0.9833**	1.0714	**0.9586**	1.0029	1.0014

TABLE X. SUMMARY OF FACTOR EFFECTS FOR ON-OFF FUNCTION(PQL)

Level \ Factor	A	B	C	D	E
Level 1	1.1983	**0.5513**	**0.3815**	1.4027	**0.5992**
Level 2	1.0000	1.0000	1.0000	**1.0000**	1.0000
Level 3	**0.6888**	0.8861	0.6694	1.0213	1.1096

Suppose that quality losses of $R_{T\text{-}ON}$, $R_{T\text{-}OFF}$ and ON-OFF function are 1:1:2, the proposed optimal parameter settings are $A_3B_1C_3D_2E_1$ corresponding with PQL_X are 0.50697, 0.89672 and 0.15230.

V. CONCLUSIONS

Robust design has become increasingly important and its applications have been extended to the decision-making process for the design of complex engineering systems. Robust design determines the most appropriate, optimizing set of parameters by minimizing variation and quality loss from the target performance of the product.

This study presents an effective method that uses Taguchi's quality loss function and SN ratio to simultaneously optimize the continuous-continuous and digital-continuous dynamic systems.

Using the PQL, transformed from the factor effects of the SN ratios, to evaluate quality, the multiple dynamic systems can be converted into a problem with a single characteristic and the optimal parameter conditions are obtained by minimizing the total PQL value. The implementation and effectiveness of the proposed approach has been illustrated using case study.

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