Interactive Particle Models in Supply Chain Management

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***Abstract*—We study on stochastic models of emergent enterprise systems. Our focus is on developing and investigating efficient analytical and numerical methodologies to evaluate the overall performance of emergent enterprise systems. As a multi-stage supply chains, an emergent enterprise system can be modeled as an interacting particle system. Under some reasonable assumptions, the overall performance can be estimated through a homogeneous Markov chain. The stationary distribution of the Markov chain can be derived analytically, as well as the overall system performance can be predicted efficiently and accurately.**

***Index Terms*—supply chain management, interactive particle, emergent enterprise system, Markov chain, multi-stage supply chain**

I. Introduction

The organizational environment in business these days is characterized by rapid change driven by fierce competition in the marketplace and evolving technologies and customer preferences. To cope with the increased uncertainty and stringent requirements placed on them, companies are teaming up, forming partnerships and alliances, joining efforts to bring better quality products, faster and cheaper to the market. As a result, the performance of any one of these companies is linked to the performance of the others, and, more specifically, to the overall performance of the system, which we call the emergent enterprise, as is observed by the final customer, the consumer. Thus, the emergent enterprise is composed of a number of independent agents that make distributed decisions but have at least one common goal: overall system performance. Consequently, each of these agents needs to estimate the performance of the distributed enterprise in order to evaluate the possible outcomes of their actions and make decisions accordingly. The main objective of this paper is to develop models of the emergent enterprise that capture the independent behavior of each of the agents involved as well as the effect of the interactions among agents, in order to accurately predict the dynamics of the organization and the performance of the system in the long run. In addition, these models will provide insight into understanding the mechanisms that result in effective alliances and organizational design.

In many ways the study of emergent enterprise systems is analogous to studying a collection of molecules or subatomic particles. Each particle has physical characteristics, similar to the capabilities of an entity in the enterprise. Particles interact with each other by exerting different force fields, similar to the interaction between organizations via transfer of material or information. Particles coalesce into groups, such as molecules or paired particles, based on these interactions, similar to the coalition formation between the organizations in an enterprise. The mass properties of the collection of particles such as temperature or pressure depend on the interactions between particles and the environmental conditions, similar to the overall behavior of the enterprise, which depends on the interactions between organizations and the operating environment.

In what follows, we model the dynamics of the supply chain as an interacting particle system and determine its limiting behavior [1]-[8]. For that purpose, we first need to define the interaction between the different agents and the decision process associated with each of them. We start by considering a simple two-stage supply chain. We later demonstrate how to extend our approach to three stages and, by similar arguments, its applicability to any number of stages.

II. Two-Stage Supply Chain Model

Under some reasonable conditions, the Two-Stage Supply Chain can be modeled as a homogeneous Markov chain with finite state space. If the buyers' ordering

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quantities are constant and independent of time, the stationary probability distributions, the limit probabilities of buyers order from suppliers, are successfully derived.

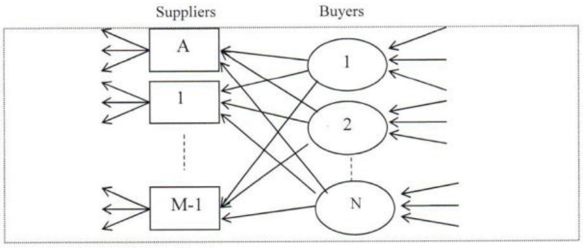

Figure 1. Two-stage supply chain model.

Consider the two-stage supply chain described in Fig. 1. Without loss of generality, we focus on the performance of a single supplier, say supplier *A*. Let *N* be the number of potential buyers of its output. The demand for buyer *i* at time *t* is a random variable, $D_i(t)$ which we assume to be independent and identically distributed over time. Given its demand and inventory policy, buyer *i* will decide the quantity $Q_i(t)$ to order from supplier *A* or any of the competing suppliers.

A buyer makes a decision to choose a supplier based on two factors: (a) its performance or utility value (based on price, quality, service, field support, etc.), and (b) its market share. For example, a company that currently orders components from supplier *A* may switch to a supplier *B* that serves a greater portion of the market or it may decide to buy from the supplier providing the best service. To account for these two aspects, we introduce parameters $\alpha_i, i=1,2,...N, 0 \le \alpha_i \le 1,$, to describe the importance placed on each factor by the corresponding buyer. Specifically, buyer *i* bases her decision at time *t* on the market share of supplier *A* with probability α_i and on the performance (or utility) of supplier *A* with probability 1- α_i . The performance rate or utility value that buyer *i* associates with supplier *A* is represented by a parameter ρ_i , $0 \le \rho_i \le 1$. We also need to model how supplier performance is affected by its overall demand: it will improve as demand increases due to economies of scale, but deteriorate when demand reaches its capacity limit. Thus, the demand from a buyer is not always an increasing function of the total market share of supplier *A*. Once the supplier reaches its peak utilization, it will become less attractive. To incorporate this effect, we assume that the attractiveness of a buyer *i* to a supplier is a concave function, $f_i(r)$, of the supplier's market share *r*. For example, $f_i(r) = C(1-(1-r/C)^2)$, where *C* is the optimum share given the supplier's capacity; $f_i(r)$ is increasing when *r* < *C*, but $f_i(r)$ is decreasing when $r \ge C$. We call buyer *i* a *Concave Buyer* if $f_i(r)$ is a concave function and a *Linear Buyer* if it is linear. Finally, we consider one additional parameter $\lambda_{i,} 0 \le \lambda_i \le 1$, which represents the rate at which buyer i would break an ongoing relationship with supplier *A*.

Our objective is to develop an analytical model that captures enough details of the decision making processes of the individual companies as well as the interaction among them, and allows us to determine the evolution and long-term stability of supply chain relationships. In the stylized analytical model we assume the overall order quantity to be known and constant over time, that is, $Q_i(t) = E[D_i(t)]$ for all *t*.

To model this problem as an interacting particle system, we define binary random variables:

$$Y_i = \begin{cases} 1, \text{if buyer } i \text{ orders from supplier } A \text{ at tim } t, \\ 0, \text{otherwise} \end{cases}$$

Note that this would be equivalent to an interacting particle system in which a site representing supplier *A* has *N* neighboring sites; there is a particle at site *i*, $i = 1,...N$ if buyer *i* favors supplier *A* or, otherwise, the site is empty.

Then, $E[Y_i(t)] = P\{Y_i(t)=1\}$ is buyer *i*'s ordering rate from supplier *A* at time *t*. The expected amount that buyer *i* orders from supplier *A* at time *t* is given by

$$Q_i^A(t) = E[Y_i(t)]Q_i(t)$$

<u>Notation Summary</u>

N: number of buyers;

$D_i(t)$: demand observed by buyer *i* at time *t;*

$Y_i(t)$: 1 if buyer *i* is purchasing from supplier *A* at time *t*, 0 otherwise;

$Q_i(t)$: overall order quantity of buyer *i* at time t (from all suppliers);

$Q_i^A(t)$: expected quantity ordered by buyer *i* to supplier *A* at time *t;*

α_i : a factor to choose supplier *A;*

ρ_i : the performance rate or utility value of supplier *A* as observed by buyer *i;*

$f_i(r)$: buyer *i*'s rate of attraction to supplier *A* based on its market share, *r;*

$\lambda_{i,}$: the rate at which buyer *i* breaks a relationship with supplier *A*;

$P(t)$: the transition matrix at step *t;*

P_t: the instant probability vector at time *t;*

p_i : the limiting probability (chance) of buyer *i* to purchase from supplier *A*.

Given these parameters and notation, we define the following transition probabilities in the system.

$$P\{Y_i(t)=1 \mid Y_i(t-1)=0, Y_j(t-1)=y_{j,} 1 \le j \le N, j \ne i\}$$
$$= \alpha_i f_i\left(\sum_{j \ne i} y_j Q_j(t-1)/Q(t-1)\right) + (1-\alpha_i)\rho_i,$$
$$P\{Y_i(t)=0 \mid Y_i(t-1)=0, Y_j(t-1)=y_j, 1 \le j \le N, j \ne i\}$$
$$= 1-\alpha_i f_i\left(\sum_{j \ne i} y_j Q_j(t-1)/Q(t-1)\right) - (1-\alpha_i)\rho_i$$

$$P\{Y_i(t)=0 \mid Y_i(t-1)=0, Y_j(t-1)=y_j, 1\le j\le N, j\ne i\}$$
$$=\alpha_i\lambda_i\left[1-f_i\left(\left[Q_i(t-1)+\sum_{j\ne i}y_jQ_j(t-1)\right]/Q(t-1)\right)\right]$$
$$+(1-\alpha_i)(1-\rho_i),$$
$$P\{Y_i(t)=1 \mid Y_i(t-1)=1, Y_j(t-1)=y_j, 1\le j\le N, j\ne i\}$$
$$=1-\alpha_i\lambda_i\left[1-f_i\left(\left[Q_i(t-1)+\sum_{j\ne i}y_jQ_j(t-1)\right]/Q(t-1)\right)\right]$$
$$-(1-\alpha_i)(1-\rho_i),$$

where $Q(t)=\sum_{j=1}^{N}Q_j(t)$ and $y_i\in\{0,1\}$. Since $Y_1(t),Y_2(t),...,Y_N(t)$ are independent given that $Y_1(t-1),Y_2(t-1),...,Y_N(t-1)$ are known, we have

$$P\{Y_j(t)=y_j^{'}, j=1,\cdots,N \mid Y_i(t-1)=y_i, i=1,...,N\}$$
$$=\prod_{j=1}^{N}P\{Y_j(t)=y_j^{'} \mid Y_i(t-1)=y_i, i=1,...,N\}.$$

$\{Y(t)=(Y_1(t),\cdots,Y_N(t)), t\ge 0\}$ is a Markov chain with a state space $Z=\{y=(y_1,\cdots,y_N), y_i\in\{0,1\}, i=1,...,N\}$, a total of 2^N elements. Let the transition probability matrix at time *t* be $P(t)$ and let the initial vector of buyer choices be $y^0=(y_1^0,...,y_N^0)$; that is, the initial probability vector P_0 is

$$P_0=(\overbrace{0,\cdots,0}^{\text{otherstates}},\overset{y^0}{\hat{1}},\overbrace{0,\cdots,0}^{\text{otherstates}})^T,$$

And the probability vector at time *t* is given by

$$P_t=P(t)P_{t-1}=\cdots=P(t)P(t-1)\cdots P(1)P_0.$$

Observe that when $\alpha_i=0$ and $\rho_i=0$, buyer *i* would never order from supplier *A*. This is true since $\alpha_i=0$ implies that buyer *i* bases her decisions solely on the utility value of the supplier, which is given by ρ_i. For convenience, we assume that α_i and ρ_i cannot be zero simultaneously. Consequently, $\{Y(t), t\ge 0\}$ is a non-singular Markov Chain with a finite state space, and homogeneous since we consider the ordering quantities of buyers to be deterministic and constant over time. Thus, there exists a stationary probability vector $\pi=(\pi_1,\cdots,\pi_{2^N})$.

$$P_t=P(t)P(t-1)\cdots P(1)P_0=P(1)^tP_0\xrightarrow{t\to\infty}\pi^T,$$

for any initial probability vector P_0. We obtain the probability vector at any time *t*. In theory, we can derive the stationary probability vector from the above Equation.

How do we calculate the market shares in long run for supplier *A?* For example, when *N=2*, the state space $Z=\{(0,0),(0,1),(1,0),(1,1)\}$. The stationary probability distribution is $\pi=(\pi_{(0,0)},\pi_{(0,1)},\pi_{(1,0)},\pi_{(1,1)})$. Therefore the chance of buyer 1 to purchase from supplier *A* in long term is

$$p_1=\lim_{t\to\infty}P(Y_1(t)=1)=\pi_{(1,0)}+\pi_{(1,1)}.$$

If the overall quantity $Q_i(t)=E[D_i(t)]$ is a constant in time *t*, say Q_i, the stationary market shares for supplier *A* from buyer 1 is

$$Q_1^A=p_1Q_1.$$

In general a more accurate model can be derived. The transition matrix may be different. Many factors are involved in deriving the transition probabilities, such as delivery time, supplier's capacity, product's price, scoring cards, and supplier's past performance.

III. Two-Stage Batch Supply Chain Model

In many cases in a supply chain system, there are capacity limits for all suppliers. Sometimes, buyers do not need all orders to be delivered at the same time in order to save inventory costs. Buyers may use different strategy to find the best (optimal) supplier, such as using auction to find a best supplier, selecting two suppliers at the same time to compete to drop the price down. Therefore a more flexible model is needed. In the assumption of the basic Two-Stage Supply Chain model, a single buyer only can order his/her all demand from one and only one supply. We consider each buyer is a batch of buyers. In the previous model, we simply replace each buyer by several buyers in theory. This model allows each buyer to order all his/her items from different buyers. Under the same conditions (assumptions), the stationary probability distributions are derived.

In the previous section in the Fig. 1, we can decompose the buy 1 into two sub buyers. In other words, the buyer 1 is a batch of two buyers. The demand observed by buyer 1 at time *t*, $D_1(t)$, also can be decomposed into two sub demands $D_{11}(t)$ and $D_{12}(t)$ corresponding to sub buyer 1-1 and buyer 1-2 with

$$D_1(t)=D_{11}(t)+D_{12}(t).$$

See the following Fig. 2 for details. Similarly we can decompose any buyer into several sub buyers.

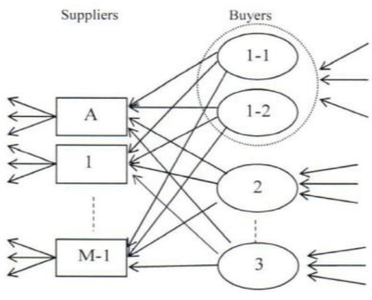

Figure 2. Two-stage supply batch chain model

Fig. 2 is a two-stage supply chain too. All results we have derived in the previous section can be applied to this section.

IV. THREE-STAGE SUPPLY CHAIN MODEL

We use the decomposition method to extend the results of the Two-Stage Supply Chain Mode to the Three-Stage Supply Chain Model. Similarly, we assume that all ordering quantities are time-independent constant. We use the two-stage supply chain model twice with decomposition. In the first sub model, we only consider manufacturers and buyers by using the results of the two-stage supply chain model, we derive the stationary distributions, which are probabilities that the stationary chance of buyers ordering from manufactures. In the second sub model, we use the results of the two-stage model again on the Suppliers and Manufactures. Here we treat all manufacturers in the three-stage supply chain model as buyers as in the two-stage supply chain model. In the two-stage model, we need to know the time-independent constant demands to derive the stationary probability distributions. The time-independent stationary demands of Suppliers are calculated by the sum of total expected demands from each Buyer. For example, we consider a single supply Wal-Mart Warehouse. Its expected demand from each buyer is the product of the stationary probability and the buyer's demand. The sum of all expected demands forms the stationary demands of the Wal-Mart Warehouse.

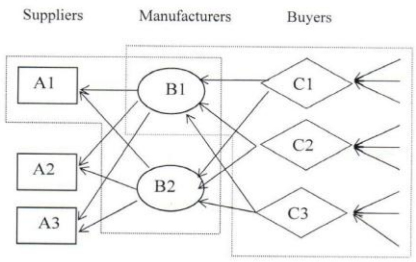

Figure 3. Three-Stage supply chain model

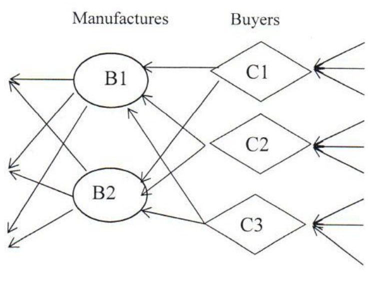

Figure 4. Two-Stage supply chain sub-model (I)

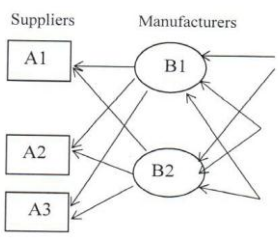

Figure 5. Two-stage supply chain sub-model (II)

V. N-STAGE SUPPLY CHAIN MODEL

Extension to the N-Stage Supply Chain Model is straightforward under the similar model assumptions. We decompose the N-Stage Supply Chain Model into N-1 sub-Two-Stage Supply Chain Models. We derive the stationary probability distributions and stationary demands from right hands side to left hand side (high level to low level).

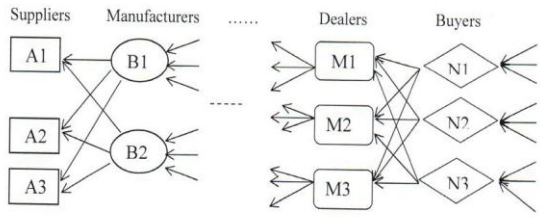

Figure 6. N-stage supply chain model

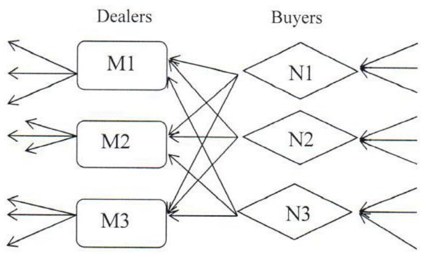

Figure 7. Two-stage supply chain model (I)

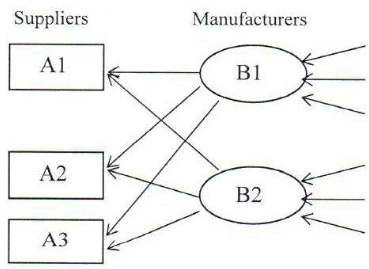

Figure 8. Two-stage supply chain model (N)

VI. NETWORK SUPPLY CHAIN MODEL

We consider a more general Network Supply Chain Model. More precisely it is a Directed (one-way) Network Supply Chain. Let us consider a similar Three-Stage Supply Chain first. In this new model, we allow all buyers of Level 4 to skip all supplier of Level 3 to order items directly from manufactures of Level 2. We construct an augmented Three-Stage Supply Chain Model. In this new augmented model, we add all Manufactures of Level 2 into Suppliers group of Level 3. This augmented Three-Stage Supply Chain Model also can be decomposed into two sub-Two-Stage Supply Chain Models. In the first sub-Two-Stage Supply Chain Model, We derive all stationary probability distributions. For example, the stationary probabilities of Wal-Mart ordering items directly from manufacture Virginia Produce Center without going through any Supplier. In the second sub-Two-Stage Supply Chain Model, we have all Manufactures on both sides (Level 2 and Level 3). We treat all manufactures in the augmented Level 3 as absorbing states. The transition probability of each manufacture on the augmented Level 3 to the same manufacture on the Level 2 is one (as defined as an absorbing state).

Similarly we extend this Three-Stage Directed Network Supply Chain Model into the N-Stage Directed Network Supply Chain Model. We construct a New Augmented N-Stage Supply Chain Model. Starting from left hand side to the right hand side, we add all components of Level 1 to Level 2.Then we add the augmented Level 2 to Level 3. Then we add the augmented Level 3 to Level 4. We repeat the same procedure. Finally we add the augmented Level N-2 to Level N-1.

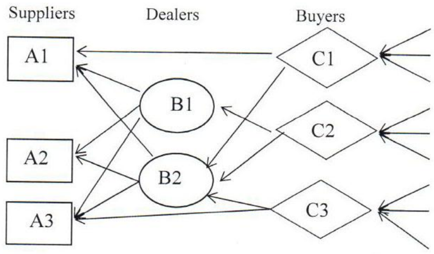

Figure 9. Three-Stage directed network supply chain model

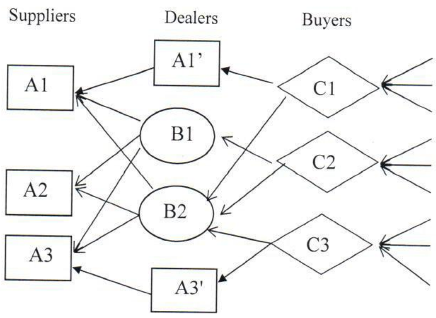

Figure 10. Three-Stage augmented supply chain model

VII. Summary and Conclusion

The development of models of the emergent enterprises capture the independent behavior of each agent involved, as well as the effect of the interactions among agents, in order to predict the performance of the system in the long run. In addition, these models will provide insight into understanding the mechanisms that result in effective alliances and organizational design.

To incorporate the interaction between various agents into our models, we formulate the problem as an interacting particle system. In many ways the study of emergent enterprise systems is analogous to studying a collection of molecules or subatomic particles. Each particle has physical characteristics, similar to the capabilities of an entity in the enterprise. Particles interact with each other by exerting different force fields, similar to the interaction between organizations via transfer of material or information. Particles coalesce into groups, such as molecules or paired particles, based on these interactions, similar to the coalition formation between the organizations in an enterprise. The mass properties of the collection of particles, such as temperature or pressure, depend on the interactions between particles and the environmental conditions, similar to the overall behavior of the enterprise, which depends on the interactions between organizations and the operating environment. Most of the research on interacting particle systems has focused on the asymptotic behavior of the system, describing the class of invariant measures for the process and determining the domain of attraction of each invariant measure. The results obtained in this area could be used to predict the long-term performance of the emergent enterprise, if modeled appropriately.

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