Fuzzy AHP-TOPSIS Strategic Theme Selection and Application

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***Abstract*—Today, many private or corporate companies have to make strategic decisions on various issues. The purpose of these decisions is to achieve either maximum efficiency or profit. In this context, two topics are especially emphasized. One is the balanced scorecard (BSC) method and the other is the multi-criteria group decision-making. While the scorecard method sets the criteria used to achieve the company's success, the multi-criteria decision-making methods evaluate the data obtained from the group decision makers. FAHP (fuzzy analytical hierarchy process) and FTOPSIS (fuzzy technique for order preference by similarity of an ideal solution) are the most recently developed methods. The purpose of our work; is to determine the themes to be established during the strategy selection process of the IETT (Istanbul Electric Tram and Tunnel) Management Company, which operates in the state-owned and public transportation sector, whose criteria are determined by the institutional report method, employing fuzzy AHP and fuzzy TOPSIS method.**

***Index Terms*—fuzzy AHP; fuzzy TOPSIS; balanced scorecard**

I. Introduction

In recent years, it has been understood how strategic management is important in not only profit-oriented private companies but also non-profit public institutions. In the strategic decision-making and planning stages, Kaplan and Norton's balanced scorecard method is used by many private companies. Initially the focus and practice of balanced scorecard is directed towards the private sector (profit-making institutions), but it also provides an excellent opportunity for the development of management in state-owned and non-profit-making institutions [1].

Balanced scorecard method examines four different dimensions in order to measure the extent to which the needs are met effectively and efficiently. These are financial dimension, customer dimension, inner process dimension, learning and innovation dimensions. Accordingly, the financial dimension [2] demonstrates whether financial performance indicators contribute to the growth of the company's strategy, practices and management's profitability. The customer dimension [3] assesses the quality of the products produced by the company at the customer's point of view. The quality of services is measured by using indicators such as error rate, rate of service fulfillment on time. The internal process dimension [3] examines what arrangements should be made within the company to meet customer expectations in a customer-focused management system. In the public sector, as in the private sector, small or large institutions have dissimilar business processes. Balanced scorecard method makes it possible to measure the results of these business processes and to select the best alternative that enables the achievement of the corporate mission and the improvement of the results. The dimension of learning and innovation [4] attaches importance to innovating the institution in order to provide the best service to the beneficiaries of the service in the public sector as it is in the private one. Thus, it is necessary to increase the skills and motivation of the employees in order to fulfill the corporate objectives. Considering these four dimensions, necessary criteria can be established in private and public institutions.

AHP (Analytic Hierarchy Processes) and TOPSIS (Technique for Order Preference Similarity to Ideal Solution) methods can be applied to many areas where criteria are set. The AHP and TOPSIS methods are based on the determination of the weights of integer scores obtained from a group of decision-makers after they have been processed through a series of calculations.

Integers are employed in classical AHP and TOPSIS methods. But people's predictions may not be expressed exactly by integer scores. In this context, fuzzy sets are used in order to make the predicted linguistic expressions more understandable and produce more reliable results.

The theory of fuzzy sets was first introduced to the literature by Zadeh [5] in 1965 and developed rapidly being used in many researches. The first fuzzy AHP study by Laarhoven and Pedrycz (1983) [6] compared the fuzzy rates defined by trapezoidal membership functions. Buckley (1985) [7] determined the fuzzy priorities of the

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comparison ratios by the trapezoidal membership function. The first fuzzy TOPSIS study was created by Chen (2000) [8] as extensions of the TOPSIS method via a system-analysis-engineer hiring process of group decision-making of software company in a fuzzy environment.

II. Fuzzy Set Theory

Definition 1: A fuzzy set $\tilde{n}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{n}}(x)$ which associates with each element x in X a real number in the interval [0,1]. The function value $\mu_{\tilde{n}}(x)$ is termed the grade of membership of x in $\tilde{n}$ [9].

Definition 2: A fuzzy set $\tilde{n}$ in the universe of discourse X is convex if and only if

$$\mu_{\tilde{n}}(\lambda x_1 + (1-\lambda)x_2) \geq min\big(\mu_{\tilde{n}}(x_1), \mu_{\tilde{n}}(x_2)\big). \quad (1)$$

For all x_1 , x_2 in X and all $\lambda \in [0,1]$, where min denotes the minimum operator [10].

Definition 3: The height of a fuzzy set is the largest membership grade attained by any element in that set. A fuzzy set $\tilde{n}$ in the universe of discourse X is called normalized when the height of $\tilde{n}$ is equal to 1 [10].

Definition 4: A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal [9].

Definition 5: The $\alpha - cut$ of fuzzy number $\tilde{n}$ is defined as $\tilde{n}^{\alpha} = \{x_i : \mu_{\tilde{n}}(x_i) \geq \alpha, x_i \in X\}$, (2)

Where $\alpha \in [0,1]$.
The symbol $\tilde{n}^{\alpha}$ represents a non-empty bounded interval contained in X, which can be denoted by $\tilde{n} = [\tilde{n}_l, \tilde{n}_u]$, $\tilde{n}_l$ *and* $\tilde{n}_u$ are the lower and upper bounds of the closed interval, respectively [9,11]. For a fuzzy number $\tilde{n}$, if $\tilde{n}_l > 0$ and $\tilde{n}_u \leq 1$ for all $\alpha \in [0,1]$, then $\tilde{n}$ is called a standardized (normalized) positive fuzzy number [12].

Definition 6: positive trapezoidal fuzzy number (PTFN) $\tilde{n}$ can be defined as (n_1, n_2, n_3, n_4) .The membership function $\mu_{\tilde{n}}(x)$ is defined as [9].

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x < n_1, \\ \frac{x-n_1}{n_2-n_1}, & n_1 \leq x \leq n_2, \\ \frac{x-n_4}{n_3-n_4}, & n_3 \leq x \leq n_4, \\ 0, & x > n_4. \end{cases} \quad (3)$$

For a trapezoidal fuzzy number $\tilde{n} = (n_1, n_2, n_3, n_4)$if $n_2 = n_3$, then $\tilde{n}$ is called a triangular fuzzy number. A non-fuzzy number r can be expressed as (r,r,r,r). By the extension principle [13], the fuzzy sum $\oplus$ and fuzzy Subtraction $\ominus$ of any two trapezoidal fuzzy numbers are also trapezoidal fuzzy numbers; but the multiplication $\otimes$ of any two trapezoidal fuzzy numbers is only an approximate trapezoidal fuzzy number. Given any two-positive trapezoidal fuzzy numbers, $\tilde{m} = (m_1, m_2, m_3, m_4), \tilde{n} = (n_1, n_2, n_3, n_4)$and a positive real number r, some main operations of fuzzy numbers $\tilde{m}$ and $\tilde{n}$ can be expressed as follows:

$$\tilde{m} \oplus \tilde{n} = [m_1 + n_1, m_2 + n_2, m_3 + n_3, m_4 + n_4], \quad (4)$$

$$\tilde{m} \ominus \tilde{n} = [m_1 - n_4, m_2 - n_3, m_3 - n_2, m_4 + n_1], \quad (5)$$

$$\tilde{m} \otimes \tilde{n} = [m_1 r, m_2 r, m_3 r, m_4 r], \quad (6)$$

$$\tilde{m} \otimes \tilde{n} \cong [m_1 n_1, m_2 n_2, m_3 n_3, m_4 n_4]. \quad (7)$$

Definition 7: A linguistic variable is a variable whose values are expressed in linguistic terms [11]. Fuzzy numbers can also represent these linguistic values.

Let $\tilde{m} = (m_1, m_2, m_3, m_4)$ and $\tilde{n} = (n_1, n_2, n_3, n_4)$ be two trapezoidal fuzzy numbers. Then the distance between them can be calculated by using the vertex method as [8].

$$d_v(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{4}[(\tilde{m}_1 - \tilde{n}_1)^2 + (\tilde{m}_2 - \tilde{n}_2)^2 + (\tilde{m}_3 - \tilde{n}_3)^2 + (\tilde{m}_4 - \tilde{n}_4)^2]} \quad (8)$$

III. Fuzzy AHP-Topsis Hybrid Model

The steps of the fuzzy AHP-TOPSIS hybrid method are as follows;
X: Set of alternatives
F: Set of criteria
$X = \{x_1, x_2, \dots, x_n\}$
$F = \{f_1, f_2, \dots, f_n\}$
Assuming k amount of decision makers. $(D_1, D_2, \dots, D_k)$

The set of criterion, F, is divided into two separate sets as F_1 *and* F_2. F_1 represents a set of benefit criteria, F_2 represents a set of cost criteria. In this case; $F_1 \cap F_2 = \emptyset$, $F_1 \cup F_2 = F$

Step 1: Pairwise comparison matrices for criteria, subcritaria and alternatives are constructed using linguistic terms.Each element $\tilde{a}_{ij}^k$ of the pairwise comparison matrix $\tilde{A}_k$ is a fuzzy number corresponding to its linguistic term. The pairwise comparison matrix is given by [7];

$$\tilde{A}_p = \begin{bmatrix} 1 & \tilde{a}_{12}^k & \dots & \tilde{a}_{1n}^k \\ \tilde{a}_{21}^k & 1 & \dots & \tilde{a}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1}^k & \tilde{a}_{n2}^k & \dots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \tilde{a}_{12}^k & \dots & \tilde{a}_{1n}^k \\ \frac{1}{\tilde{a}_{21}^k} & 1 & \dots & \tilde{a}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\tilde{a}_{n1}^k} & \frac{1}{\tilde{a}_{n2}^k} & \dots & 1 \end{bmatrix} \quad (9)$$

$$\bar{A} = (\tilde{\bar{a}}_{ij})_{nxn},$$

TABLE I. Definition and Interval Fuzzy Scales of the Linguistic Variable (G.Zheng et al.2012)

Trapezoidal Interval Type-1 Fuzzy Scales.	
Equally important (E)	(1,1,1,1)
Weakly important (W)	(2, 5/2,7/2, 4)
Essentially important (ES)	(4, 9/2, 11/2, 6)
Very strongly important (VS)	(6,13/2,15/2, 8)
Absolutely important (A)	(8,17/2,9, 9)

The geometric mean of k. fuzzy set is calculated as follows;

$$\bar{A} = \left(\tilde{a}_{ij}^1 \otimes \tilde{a}_{ij}^2 \otimes \dots \otimes \tilde{a}_{ij}^k\right)^{\frac{1}{k}} = \sqrt[p]{\tilde{a}_{ij}^1 \otimes \tilde{a}_{ij}^2 \otimes \dots \otimes \tilde{a}_{ij}^k} \quad (10)$$

For the evaluation procedure, the linguistic terms given in Table I are used.

Step 2: The geometric mean of each line is calculated and then the normalization process is applied on the fuzzy weights. The geometric mean of each line is calculated as follows [7];

$$\tilde{r}_i = (\tilde{a}_{i1} \otimes \tilde{a}_{i2} \dots . \otimes \tilde{a}_{in})^{1/n} \quad (11)$$

$$\widetilde{w}_i = \tilde{r}_i \otimes (\tilde{r}_1 + \tilde{r}_2 + \dots + \tilde{r}_n)^{-1} \quad (12)$$

Here, $\tilde{a}_{in}$ is the linguistic evaluation of the criterion compared to the n. criterion, $\tilde{r}_i$ is the geometric mean value that is calculated by comparing the measure with all the criteria.

Step 3: Calculation of global weights for each sub criterion;

$$\widetilde{w}_{ijg} = \widetilde{w}_{ijl} \otimes \widetilde{w}_i \,, 1 \le i \le m. and\ 1 \le j \le n. \quad (13)$$

Step 4: For p. decision maker, a decision matrix of Y_p and mean decision matrix $\bar{Y}$ are created [14].

$$Y_P = \left(\tilde{f}_{ij}^P\right)_{mxn} = \begin{array}{c} \\ \tilde{f}_{11}^P \\ \tilde{f}_{21}^P \\ \vdots \\ \tilde{f}_{m1}^P \end{array} \begin{array}{c} \begin{array}{ccccc} X_1 & X_2 & & \dots & X_n \end{array} \\ \begin{bmatrix} \tilde{f}_{11}^P & \tilde{f}_{12}^P & \dots & \tilde{f}_{1n}^P \\ \tilde{f}_{21}^P & \tilde{f}_{22}^P & \dots & \tilde{f}_{2n}^P \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{f}_{m1}^P & \tilde{f}_{m2}^P & \dots & \tilde{f}_{mn}^P \end{bmatrix} \end{array} \quad (14)$$

$$\bar{Y} = \left(\tilde{f}_{ij}\right)_{m \times n} , \quad (15)$$

Where $\tilde{f}_{ij} = \left(\frac{\tilde{f}_{ij}^1 \oplus \tilde{f}_{ij}^2 \oplus \dots \oplus \tilde{f}_{ij}^k}{k}\right)$, $\tilde{f}_{ij} = \left(a_{ij,} b_{ij,} c_{ij,} d_{ij}\right)$ are trapezoidal fuzzy numbers.

The linguistic terms used in the application of the fuzzy numbers to be used in the Fuzzy TOPSIS method are shown in Table II.

TABLE II. LINGUISTIC TERMS OF WEIGHTS OF THE ATTRIBUTES AND THEIR CORRESPONDING FUZZY SETS (CHEN 2000).

Trapezoidal Interval Type-1 Fuzzy Scales	
Very Low (VL)	(0, 0, 0, 0.1)
Low (L)	(0, 0.1, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.7, 0.9)
High (H)	(0.7, 0.9, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0, 1.0)

Step 5: A weighted decision matrix

$$\bar{Y}_w = \left(\tilde{v}_{ij}\right)_{mxn} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \begin{array}{c} \begin{array}{ccccc} X_1 & X_2 & & \dots & X_n \end{array} \\ \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \dots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \dots & \tilde{v}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \dots & \tilde{v}_{mn} \end{bmatrix} \end{array} \quad (16)$$

$$\tilde{v}_{ij} = \widetilde{w}_{ijg} \otimes \tilde{f}_{ij}, 1 \le i \le m. and\ 1 \le j \le n. \quad (17)$$

Step 6: The fuzzy positive-ideal solution (FPIS,A^*) and the fuzzy negative-ideal solution (FNIS,A^-) are defined as [15]:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*), A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \quad (18)$$

Where $\tilde{v}_j^* = (1,1,1,1)$ and $\tilde{v}_j^- = (0,0,0,0)$, $j = 1,2, \dots, n$.

$$d_i^* = \sum_{j=1}^{n} d_v\left(\tilde{v}_{ij}, \tilde{v}_j^*\right), \quad i = 1,2, \dots, m, \quad (19)$$

$$d_i^- = \sum_{j=1}^{n} d_v\left(\tilde{v}_{ij}, \tilde{v}_j^-\right), \quad i = 1,2, \dots, m, \quad (20)$$

Step 7: The closeness coefficient is determined [15].

$$CC_i = \frac{d_i^-}{d_i^- + d_i^*} \quad (21)$$

Step 8: Alternatives are ranked in decreasing order. The larger the value of CC_i the higher the preference of the alternative [15].

IV. NUMERICAL EXAMPLES

In this section, examples are presented demonstrating the decision-making process of the fuzzy multi-feature set of the proposed method.

Alternatives are strategically selected by the IETT's expert staff, taking into consideration of the annual, 5-year and 10-year development plans and following the latest trends and investments in the world population, based on the previous knowledge, experience, and strategies of the IETT. The experts in this field constitute the directors of the departments designated by the relevant authorities. In this context, 7 decision makers and 12 alternatives are foreseen.

IETT refers to Kaplan and Norton's Balanced Scorecard Method. The main and sub-criteria were selected by expert staff by analyzing the consistency with each other in accordance with the institutional report method.

Accordingly, 12 sub-criteria were selected as the most appropriate for the 4 main criteria and institution specified in the Balanced Scorecard.

The linguistic terms used in the TOPSIS method are shown in Table II as "Very Low" (VL), "Low" (L), "Medium Low" (ML), "Medium" (M), "Medium High" (MH), "High" (H), "Very High" (VH). The linguistic terms used in the AHP method are shown in Table I as "Certainly Strong"(CS)," Very Strong "(VS)," Quite Strong "(QS)," Somewhat Strong "(SS), "Equal" (E).It is aimed to figure out the most suitable option for investment from among the twelve themes for İETT. There are seven decision makers, four main criteria, twelve sub criteria and twelve alternatives.

Decision makers are denoted as D_1, D_2 , D_3, D_4, D_5, D_6 and D_7, alternatives as a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11} and a_{12}, criteria as, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11} and k_{12}. Alternatives include "Quality of Service" (a_1) , "New Product and Service Development" (a_2), "Innovation" (a_3), "Effective and

Efficient Processes" (a_4) , "Leadership and Communication" (a_5), "Holistic Leadership" (a_6), "Agility in Service" (a_7), "Strong Financial Structure" (a_8) , "Sustainable Services" (a_9) , Mankind-Environment-Profit (a_{10}), "Business and Passenger Safety" (a_{11}), "Road and passenger safety" (a_{12}). The main criteria are "Customer" (k_{a1}), "Finance" (k_{a2}), "Internal Process" (k_{a3}) , "Learning and Development" (k_{a4}), and the sub-criteria are "Customer Satisfaction" (k_1), "New Customer" (k_2), "Customer Loyalty" (k_3), "Cost" (k_4)," Income" (k_5), "Financial Sustainability" (k_6), "Quality" (k_7), "Productivity" (k_8), "Activity" (k_9) , "Employee Qualification" (k_{10}) , "Information System Competency" (k_{11}), "Motivation Authorization and Adaptation" (k_{12}).

Consistency ratios are calculated based on the data obtained through questionnaires of each decision maker, by creating binary comparison matrices as in Table III and by being dependent on previously given linguistic terms. In our numerical example, all decision tables are consistent. Consistency ratios are less than 0,1.

Step 1: Table III shows the binary comparison matrices between the main criterion and the sub criteria in the hierarchical system.

TABLE III. THE BINARY COMPARISON MATRIX OF THE MAIN CRITERIA AND SUB CRITERIA FOR THE D1 DECISION MAKER

Main and Sub criterion comparison matrix				
Main Criterion Matrix				
	k_{a1}	k_{a2}	k_{a3}	k_{a4}
k_{a1}	E	ES	VS	ES
k_{a2}	1/ES	E	W	1/W
k_{a3}	1/VS	1/W	E	1/W
k_{a4}	1/ES	W	W	E

Table IV shows the calculation of the decision matrix by taking the geometric mean of the binary comparison matrices of the main criterion and the sub criteria obtained from the evaluation of the relevant decision makers.

TABLE IV. THE BINARY COMPARISON OF GEOMETRIC MEAN DECISION MATRICES

Binary Comparison Of Geometric Mean Decision Matrices		
	k_{a1}	k_{a2}
k_{a1}	(1.00,1.00,1.00,1.00)	(3.48,4.01,5.05,5.57)
k_{a2}	(0.18,0.20,0.25,0.29)	(1.00,1.00,1.00,1.00)
k_{a3}	(0.17,0.18,0.20,0.21)	(0.34,0.37,0.47,0.54)
k_{a4}	(0.14,0.14,0.17,0.18)	(0.33,0.37,0.46,0.52)
	k_{a3}	k_{a4}
k_{a1}	(4.76,5.10,5.67,5.90)	(5.47,6.04,6.96,7.31)
k_{a2}	(1.84,2.11,2.67,2.97)	(1.92,2.20,2.74,3.02)
k_{a3}	(1.00,1.00,1.00,1.00)	(1.22,1.41,1.79,2.00)
k_{a4}	(0.50,0.56,0.71,0.82)	(1.00,1.00,1.00,1.00)

TABLE V. NORMALIZED OF MAIN CRITERIA

Normalization Of Main Criteria
Main Criterion Matrix
k_{a1} (0.48,0.55,0.71,0.81)
k_{a2} (0.14,0.16,0.22,0.26)
k_{a3} (0.08,0.09,0.12,0.14)
k_{a4} (0.06,0.07,0.09,0.11)

Step 2: Employing Table IV and Equation 11, the geometric mean of each line is calculated as shown in $\tilde{r}_1$. $\tilde{r}_1$, $\tilde{r}_2$, $\tilde{r}_3$, $\tilde{r}_4$, $\tilde{r}_5$, $\tilde{r}_6$, $\tilde{r}_7$, $\tilde{r}_8$, $\tilde{r}_9$, $\tilde{r}_{10}$, $\tilde{r}_{11}$, $\tilde{r}_{12}$, $\tilde{r}_{13}$, $\tilde{r}_{14}$, $\tilde{r}_{15}$ and $\tilde{r}_{16}$ are calculated by the same method.

$$\tilde{r}_1 = [\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \tilde{a}_{13} \otimes \tilde{a}_{14} \otimes]^{\frac{1}{4}}$$
$$= [(1,1,1,1) \otimes (3.48,4.01,5.05,5.57)$$
$$\otimes (4.76,5.10,5.67,5.90) \otimes (5.47,6.04,6.96,7.31)]^{\frac{1}{4}}$$
$$= (3.09,3.33,3.76,3.94)$$

Employing Table IV and Equation 12, normalization over fuzzy weights is calculated as shown in in $\widetilde{w}_1$ *and* $\widetilde{w}_{l1}$. Normalization of main criteria $\widetilde{w}_1$, $\widetilde{w}_2$, $\widetilde{w}_3$, $\widetilde{w}_4$ and sub criteria $\widetilde{w}_{l1}$, $\widetilde{w}_{l2}$, $\widetilde{w}_{l3}$, $\widetilde{w}_{l4}$, $\widetilde{w}_{l5}$, $\widetilde{w}_{l6}$, $\widetilde{w}_{l7}$, $\widetilde{w}_{l8}$, $\widetilde{w}_{l9}$, $\widetilde{w}_{l10}$, $\widetilde{w}_{l11}$, $\widetilde{w}_{l12}$ are calculated by the same method.

Step 3: Employing Table V and Equation 13, the global weight is calculated as shown in $\widetilde{w}g_1$. Table VI displays $\widetilde{w}g_1$, $\widetilde{w}g_2$, $\widetilde{w}g_3$, $\widetilde{w}g_4$, $\widetilde{w}g_5$, $\widetilde{w}g_6$, $\widetilde{w}g_7$, $\widetilde{w}g_8$, $\widetilde{w}g_9$ $\widetilde{w}g_{10}$, $\widetilde{w}g_{11}$, and $\widetilde{w}g_{12}$.

$$\widetilde{w}_{g1} = \widetilde{w}_1 \otimes \widetilde{w}_{l1}$$
$$= (0.48,0.55,0.71,0.81) \otimes (0.43,0.49,0.65,0.74)$$
$$= (0.21,0.27,0.46,0.60)$$

TABLE VI. GLOBAL WEIGHT OF SUB CRITERIA

Global Weight Of Sub Criteria	
For k_{a1} Matrix	**For k_{a3} Matrix**
k_1 (0.21,0.27,0.46,0.60)	k_7 (0.03,0.04,0.07,0.09)
k_2 (0.08,0.10,0.17,0.22)	k_8 (0.01,0.02,0.03,0.05)
k_3 (0.09,0.11,0.18,0.24)	k_9 (0.02,0.02,0.04,0.05)
For k_{a2} Matrix	**For k_{a4} Matrix**
k_4 (0.03,0.03,0.06,0.09)	k_{10} (0.02,0.02,0.04,0.06)
k_5 (0.02,0.02,0.04,0.06)	k_{11} (0.01,0.02,0.03,0.04)
k_6 (0.06,0.08,0.16,0.22)	k_{12} (0.01,0.02,0.03,0.04)

TABLE VII. EVALUATION OF ALTERNATIVES ACCORDING TO THE CRITERIA BY RELEVANT DECISION MAKERS

Attributes	Alternatives	Decision-Makers						
		D1	D2	D3	D4	D5	D6	D7
k_1	A1	VH	VH	VH	VH	VH	VH	VH
	A2	M	ML	H	ML	H	MH	MH
	A3	MH	ML	MH	L	H	M	H
	A4	L	M	MH	ML	M	ML	ML
	A5	ML	H	M	L	L	M	VL
	A6	MH	MH	M	L	ML	L	VL
	A7	M	MH	VH	H	MH	MH	M
	A8	VL	M	MH	M	M	ML	ML
	A9	M	H	MH	MH	M	H	H
	A10	MH	MH	MH	ML	M	VH	MH
	A11	H	H	H	H	H	H	H
	A12	VH	VH	MH	VH	VH	H	H

Step 4: A decision matrix is established for each decision maker based on the data in Table VII and on the Equation 14 and 15. Decision matrices are denoted by Y_1, Y_2, Y_3, Y_4, Y_5, Y_6 and Y_7, alternatives by a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11} and a_{12}. $\bar{Y}$ is the average decision matrix.

Based on the Equation 15, the mean decision matrix $\bar{\boldsymbol{Y}}$ can be constructed. It is calculated as follows:

$$\tilde{f}_{11} = (\ 0.90,1.00\ ,1.00\ ,1.00\ ),$$
$$\vdots$$

$\tilde{f}_{112} = (\ 0.79\ ,0.93\ ,0.93\ ,0.99\ )$

Step 5: Based on Equation 16 and 17, a weighted decision matrix $\bar{Y}_w$ is constructed. It is demonstrated as follows;

$\tilde{v}_{11} = (0.18,0.27,0.46,0.60)$

$\vdots$

$\tilde{v}_{112} = (0.16,0.25,0.43,0.59)$

Step 6: Based on Equation 18, 19 and 20, the fuzzy positive-ideal solution ($FPISA^*$) and the fuzzy negative-ideal solution ($FNISA^-$) are defined as:

$\boldsymbol{d_i^*}\ (\boldsymbol{a_1}) = 0.64 \qquad \boldsymbol{d_i^-}(\boldsymbol{a_1}) = 0.41$

$\vdots \qquad\qquad\qquad \vdots$

$\boldsymbol{d_i^*}(\boldsymbol{a_{12}}) = 0.66 \qquad \boldsymbol{d_i^-}(\boldsymbol{a_{12}}) = 0.39$

A^* =(11.05, 11.25, 11.27, 11.40, 11.51, 11.46, 11.26, 11.38, 11.20, 11.30, 11.18, 11.17).

A^- =(1.09, 0.89, 0.87, 0.72, 0.60, 0.65, 0.88, 0.74, 0.94, 0.84, 0.97, 0.97).

Step 7: Using the Equation 21, the proximity coefficient $C(a_j)$ of a_j is calculated.

Here, $1 \leq j \leq 12$.

$C(a_1) = 0.0898$, $C(a_2) = 0.0729$, $C(a_3) = 0.0713$, $C(a_4) = 0.0594$, $C(a_5) = 0.0495$, $C(a_6) = 0.0535$, $C(a_7) = 0.0727$, $C(a_8) = 0.0608$, $C(a_9) = 0.0778$, $C(a_{10}) = 0.0693$, $C(a_{11}) = 0.0796$, $C(a_{12}) = 0.0798$

Step 8: The result is as follows the result is as follows $C(a_1) > C(a_{12}) > C(a_{11}) > C(a_9) > C(a_2) > C(a_7) > C(a_3) > C(a_{10}) > C(a_8) > C(a_4) > C(a_6) > C(a_5)$.

According to the results, Quality of Service is the most important theme for transportation. After that, Road and passenger safety, Business and Passenger Safety, Sustainable Services comes respectively. The results show that, Quality a Road and passenger safety, Business and Passenger Safety, Sustainable Services and Safety comes first for public transportation. On the other hand, Sustainability is the important and popular theme in Metropol public transportation services.

V. Conclusion

This study aimed to determine the selection of the themes which are included in the 5-year development plans and which are intended to be invested, by using multi-criteria decision-making method, with reference to Balanced Scorecard-BSC used in IETT. A hybrid approach based on AHP and TOPSIS methods has been used in fuzzy environment. In our study, the scale for regulating the appropriate criteria in the theme selection was determined by the balanced scorecard method proposed by the literature. Balanced Scorecard Method (BSC) for main and sub-criteria, the FAHP method is used to determine the importance of the main criteria and sub-criteria, and the FTOPSIS method is used to rank the themes to be invested. The most important feature distinguishing this study is that this method has not been utilized before in the strategic decision-making stage of any transportation company. This method has been proposed as an approach in which decision makers' preferences are better modeled since the data received from each decision-maker is evaluated as a linguistic term. The data were obtained by questionnaires applied to expert decision makers who have authority and knowledge. The selection of the themes to be invested in IETT is carried out within a strategic framework by the SWOT analysis, taking into account the opinions of the expert decision makers. This study targets that the consistency between the data obtained from the expert decision makers is audited and the leader determines the themes to be invested and contributes to the strategic and scientific decisions of the leader.

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