Portfolio Optimization by Fuzzy Interactive Genetic Algorithm

Masato Sasaki, Anas Laamrani, Mitsuo Yamashiro, Chalew Alehegn, and Ariel Kamoyedji
Ashikaga Institute of Technology/Division of Systems and Information Engineering, Ashikaga, Japan
Email: sasaki.masato@v90.ashitech.ac.jp, {anas.laamrani, yamashiro.mitsuo, chalewpsfkaizen, cyberntique}@gmail.com

***Abstract*—In this paper, we present a Portfolio optimization method based on Interactive Genetic Algorithm and a Fuzzy satisfaction function. Portfolio optimization is a formal mathematical approach to making investment decisions across a collection of financial instruments or assets. We will be using the classical approach, known as modern portfolio theory (MPT), that involves categorizing the investment universe based on risk (standard deviation) and return, and then choosing the mix of investments that achieve a desired risk versus return tradeoff. Genetic algorithms are stochastic search algorithms inspired by biological phenomena of genetic recombination and natural selection. They simulate the evolution of string individuals encoding candidate solutions to a given problem. Genetic algorithms proved robust and efficient in finding near-optimal solutions in complex problem spaces. They are usually exploited as an optimization method, suitable for both continuous and discrete optimization tasks. We present in our proposed method an Interactive Genetic Algorithm, since it is difficult to introduce a fitness function for this kind of problem, and we will exploit instead the user/expert knowledge by interacting with our method. Finally, we will discuss and evaluate the proposed solutions by using a Fuzzy satisfaction function that takes into account the investor's subjective preference toward risk and/or return.**

***Index Terms*—interactive genetic algorithm, portfolio optimization, decision making, fuzzy satisfaction**

I. Introduction

The electronic financial market is fast emerging, the financial products endure substantial transformation with the rapid introduction of internet technologies in the financial sector. The financial portals now allow investors to get real time quotes of stock-indices, to track their evolution, to invest in mutual funds. Each of these funds has different characteristics and exhibit a different performance (expected return, risk profile) [1]. Investors who decide to invest has to consider a Portfolio optimization based on his/her subjective preferences toward the risk and/or return. However, the number of combinations of the investment plans is very huge, it is generally difficult to find the best compromised solution. Genetic Algorithm (GA) is evaluated as an excellent heuristic method for such kind of NP hard problems.

Interactive GA (iGA) is superior to the GA in the point that is able to reflect the decision maker(DM)'s subjective preferences for the real world problems. An iGA is defined as a GA that uses human evaluation [2]. These algorithms belong to a more general category of interactive evolutionary computation. The main application of these techniques includes domains where it is hard or rather impossible to design a computational fitness function, for example, evolving images [3], music [4], various artistic designs [5], and forms to fit a user's aesthetic preferences [6].

In this paper, we propose a method for supporting investor's judgement based on Portfolio optimization using iGA that present the best compromised solution set that satisfy Pareto optimality and investor's subjective preference structures toward risk and/or return such as "risk seeking", "risk averse" and "risk neutral" by applying the interactive phase toward investors. Also, to reduce the investors' burden in this interactive phase, the cluster analysis method is proposed to narrow down the solution space in Pareto frontier which satisfy the investors' preference structure. To apply iGA for portfolio optimization problems, we propose a two-layered chromosome representation. The first layer represents index set that figure candidacy brands of investments. The second layer represents the amount of investment for each brand which satisfy the total budget of investment.

Further, we develop an interactive system that present investment plan suitable for the investors based on his/her preference structure and show the effectiveness of the system, although we take into consideration the investor's subjective preference toward risk and/or return, it is unreasonable to express the investor's preferences as 100% risk seeking or 100% risk averse, that's why we use a Fuzzy Portfolio model, by introducing a fuzzy membership function [7], that can evaluate the proposed solution and indicate the investor's satisfaction.

II. Portfolio Problem

A. Portfolio Optimization

A portfolio is a grouping of financial assets such as stocks, bonds and cash equivalents, as well as their funds counterparts, including mutual, exchange-traded and closed funds. Portfolios are held directly by investors

Manuscript received March 7, 2018; revised August 6, 2018.

doi: 10.18178/joams.6.3.124-131

and/or managed by financial professionals. Prudence suggests that investors should construct an investment portfolio in accordance with risk tolerance and investing objectives [8].

When determining a proper asset allocation, one aims at maximizing the expected return and minimizing the risk. This is an example of a multi-objective optimization problem: more "efficient solutions" are available and the preferred solution must be selected by considering a tradeoff between risk and return. In particular, a portfolio A is dominated by another portfolio A' if A' has a greater expected gain and a lesser risk than A [9].

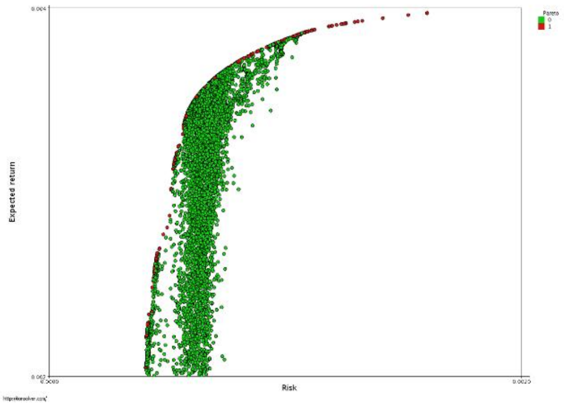


Figure 1. Risk/return plot and Pareto-optimal portfolios (in red)

B. Portfolio's Risk and Return

Modern portfolio theory assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists that has better expected returns.

C. Fuzzy Portfolio Model

Since the solution that will be selected in the portfolio is a Pareto optimal solution on the Pareto frontier, it is reasonable to formulate the portfolio problem as a multi objective problem by maximizing the expected return and minimizing the risk. The Fuzzy Portfolio model is a model that considers investor satisfaction, and applies the fuzzy concept to the target value for expected return and risk, we will use this model as an objective function to evaluate the solutions. Investors should set a sufficient level to indicate the required level of satisfaction, the minimum degree of accomplishment, and the degree of achievement to the targeted expected return and risk. This creates membership functions for targeted expected return and risk [7].

The membership function's expression is obtained by applying the sigmoid function:

$$f(x) = \frac{1}{1+exp(-\alpha x)} \quad (1)$$

The fuzzy portfolio model:

$$\begin{aligned} &\text{maximize } \lambda \\ &\text{subject to } \lambda + \exp\big(\alpha_V(V(x) - V_M)\big)\,\lambda \le 1 \quad (2) \\ &\lambda + exp\big(-\alpha_E(E(x) - E_M)\big)\,\lambda \le 1 \\ &\textstyle\sum_{i=1}^{n} x_i = 1 \end{aligned}$$

$\lambda,\ x_i \geq 0 \quad (i = 1,2, \quad ,n)$

λ: Satisfaction level of the solution

V_M: Risk value when satisfaction $\lambda = 0.5$

E_M: Expected return value for satisfaction $\lambda = 0.5$

α_E, α_v: Shape parameters of the membership function

D. Pareto Optimization

Pareto optimization is an area of multiple criteria decision making that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously.

For a nontrivial multi-objective optimization problem, no single solution exists that simultaneously optimizes each objective. In that case, the objective functions are said to be conflicting, and there exists a (possibly infinite) number of Pareto optimal solutions. A solution is called nondominated, Pareto optimal, Pareto efficient or noninferior, if none of the objective functions can be improved in value without degrading some of the other objective values. Without additional subjective preference information, all Pareto optimal solutions are considered equally good (as vectors cannot be ordered completely). The goal may be to find a representative set of Pareto optimal solutions (Fig. 1), and/or quantify the trade-offs in satisfying the different objectives, and/or finding a single solution that satisfies the subjective preferences of a human decision maker (DM) [10]-[11].

III. Cluster Analysis

A. Cluster Analysis

Cluster analysis or clustering is the task of grouping a set of objects in such a way that objects in the same group (called a cluster) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory data mining, and a common technique for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval, bioinformatics, data compression, and computer graphics.

B. K-means Clustering

In this paper, we use k-means algorithm (Fig. 2), k-means clustering aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean, serving as a prototype of the cluster. This results in a partitioning of the data space into Voronoi cells [12].

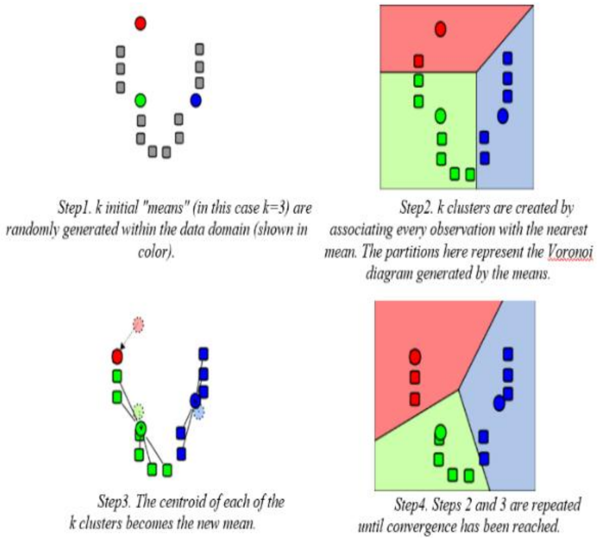


Figure 2. K-means Algorithm

IV. Interactive Genetic Algorithm

A. Genetic Algorithm

In computer science and operations research, a genetic algorithm (GA) (Fig. 3) is a metaheuristic inspired by the process of natural selection. GAs are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection [13].

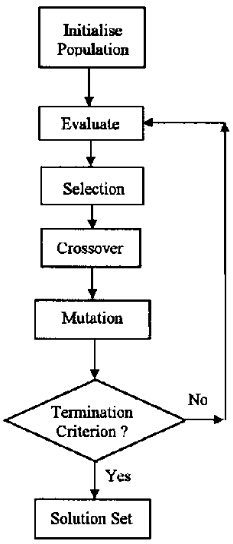


Figure 3. Genetic algorithm outline

In a GA, a population of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem is evolved toward better solutions. Each candidate solution has a set of properties (its chromosomes or genotype) which can be mutated and altered; traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible [14].

The evolution usually starts from a population of randomly generated individuals, and is an iterative process, with the population in each iteration called a generation. In each generation, the fitness of every individual in the population is evaluated; the fitness is usually the value of the objective function in the optimization problem being solved. The more fit individuals are stochastically selected from the current population, and each individual's genome is modified (recombined and possibly randomly mutated) to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population.

B. Interactive Genetic Algorithm

Interactive Genetic Algorithm (iGA) is a general term for methods of Genetic Algorithm that use human evaluation (Table 1). Usually human evaluation is necessary when the form of fitness function is not known (for example, visual appeal or attractiveness; as in Dawkins, 1986) or the result of optimization should fit a particular user preference (for example, taste of coffee or color set of the user interface) [15].

TABLE I. Comparison between IGA and GA

System	Sequences	Innovator	Selector
Interactive genetic algorithm	data	computer	human
Genetic algorithm	data	computer	human

An iGA is defined as a GA that uses human evaluation. These algorithms belong to a more general category of Interactive evolutionary computation. The main application of these techniques includes domains where it is hard or impossible to design a computational fitness function, for example, evolving images, music, various artistic designs and forms to fit a user's aesthetic preferences. Interactive computation methods can use different representations, both linear (as in traditional genetic algorithms) and tree-like ones (as in genetic programming).

The number of evaluations that iGA can receive from one human user is limited by user fatigue which was reported by many researchers as a major problem. In addition, human evaluations are slow and expensive as compared to fitness function computation. Hence, one-user iGA methods should be designed to converge using a small number of evaluations, which necessarily implies very small populations. Several methods were proposed by researchers to speed up convergence, like interactive constrain evolutionary search (user intervention) or fitting user preferences using a convex function. iGA human-computer interfaces should be carefully designed in order to reduce user fatigue. There is also evidence that the addition of computational agents can successfully counteract user fatigue [16].

However, iGA implementations that can concurrently accept evaluations from many users overcome the limitations described above.

V. Proposed Method

In the proposed solution of this paper (Fig. 4), we apply iGA by allowing the user/investor to select preferred solutions, then we introduce a fuzzy satisfaction level in the evaluation function of the portfolio problem, that selects the optimal portfolio, that satisfies all the satisfaction levels of multiple investors who have more than one objective function, it is a solution that proposes the most satisfying investment plans for the investors [17].

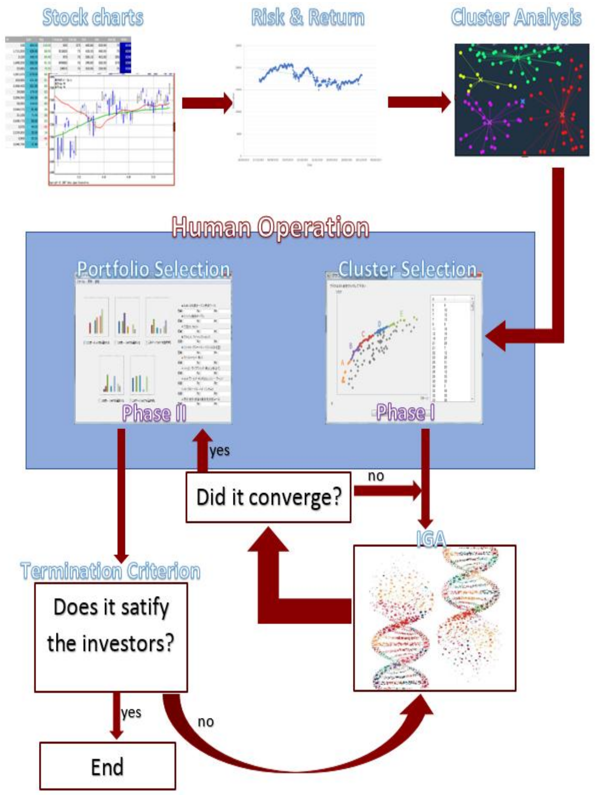


Figure 4. The diagram of the proposed solution

The steps of the proposed solution for the portfolio problem (Fig. 5) are shown below:

Step 1: Calculate Risk and Return

Import actual investment target data (historical data of the stock market), and calculate estimated return value of each investment target by using linear regression, and calculate risk by using the variance formula, then classify each stock into 5 categories (Table 2).

Step 2: Create a Pareto chart

Create a scatter plot using the risk as x axis and the return as y axis, then we will have a Pareto chart.

Step 3: Cluster Analysis

Investment targets on the scatter plot will be subjected to cluster analysis by the k-means method and will be grouped into five clusters.

Step 4: Cluster Selection

Three investment groups from five investment target groups will be selected by the investors based on priorities (phase I).

Step 5: Genetic Operation

The investment plan is acquired based on the priority of the cluster determined in step 3, a portfolio chromosome is generated using investment targets of the selected three clusters, and genetic manipulation by GA is performed.

Step 6: Convergence Test

When the evaluation value does not change from a certain value or more, it means that it's converging, then we proceed to the selection operation.

Step 7: Portfolio Selection

We present 5 portfolio chromosomes generated in Step 4 to the user, so he can select two of them (phase II).

Step 8: Evaluation

Evaluate the portfolio with the sum of three fuzzy satisfaction levels.

Step 9: Termination Criterion

When a portfolio satisfies the investors, the process end, and a final investment plan is presented. If the level of satisfaction is not enough, we return to step 4 and the process repeats.

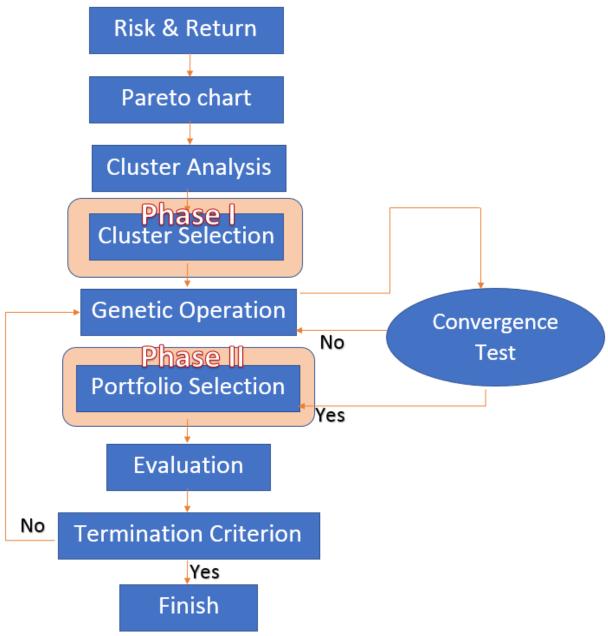


Figure 5. Flow chart of the proposed solution

A. *Model of the Problem*

To optimize the portfolio problem by applying iGA, we first build a model of the problem. In general, the optimization problem is defined by three things, the initial set of candidate solutions, constraint and objective function. On the other hand, the basic components of iGA are chromosome and evaluation function. When building a model of the portfolio problem for iGA, the candidate solution (phenotype) is associated with the chromosome (genotype) and the objective function is associated with the evaluation function, but the constraint is incorporated in the chromosome, or by adding it as a penalty function to the evaluation function (Fig. 6).

In the proposed solution, the objective function is determined as maximization of fuzzy satisfaction (fuzzy decision), and the constraint condition will be selected from the Pareto solution by Pareto ranking.

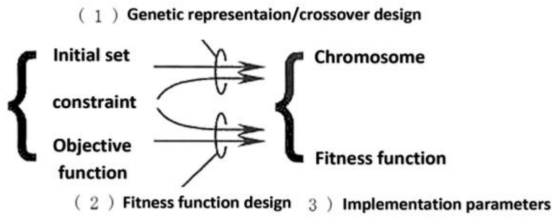


Figure 6. Design concept of iGA for the optimization problem

B. *Chromosome Representation and Genetic Operations*

In iGA of the proposed solution, when expressing the portfolio as a gene, we propose a two-layered chromosome representation as shown in Fig. 7, First, the information shows what kind of stocks, the investment target located at each locus represents, the index information is added to the stock each time we read the investment target data, and the chromosome is configured by referring to the index (Fig. 8).

In order to eliminate the difference due to the position of the chromosome, values are determined from random positions rather than in order, and when the values of the chromosome are duplicated (referring to the same index information), a mutation operation is performed. In the second layer, the investment amount for each investment target that constitutes the portfolio is used as the value of the chromosome, adjusted by satisfaction so that the total chromosome value falls within the investor's funds. Chromosome's value means the investors will have to invest 10,000 yen for every 1 step, the gene value shows the investment amount and is between 0 and K, and the second and subsequent gene values are determined from K.

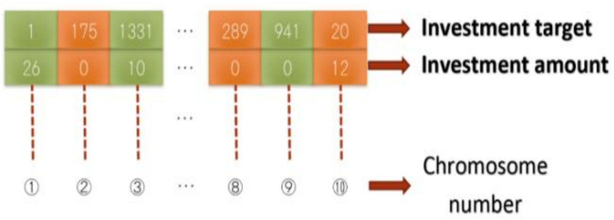


Figure 7. Two layered chromosome representation

Below are the steps showing how to determine the values of the second layer of the chromosome using the investment amount as an allele.

Step 1: Set budget K

Investors set the budget they possess K.

Step 2: Determining the value of the allele

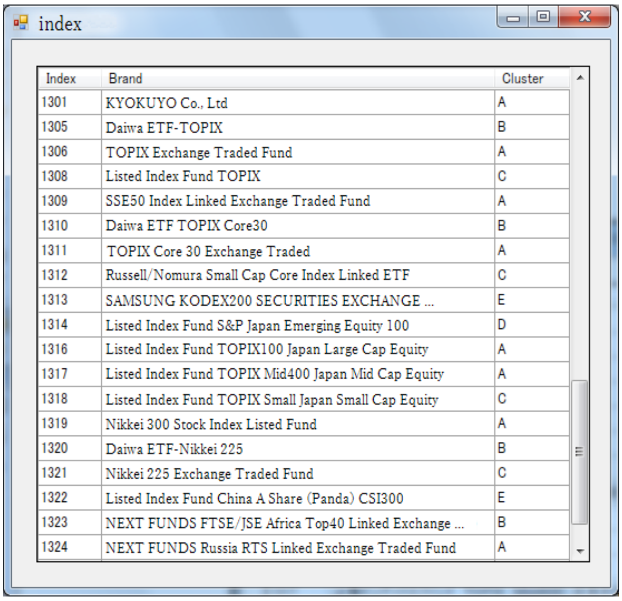
Pick up a random value (allele) of the second layer of one of the chromosomes showing the investment, the allele will take a value from 0 to K, with K being the maximum value determined in Step 1. In order to eliminate the difference due to the position of information, the position will be determined randomly.

Step 3: Repeat the process

The position of the locus is randomly determined, and the allele is randomly determined taking the value: 0～$(K-\sum_{i=1}^{n} K_i)$

Step 4: Terminal criterion

When K = 0 or the number of genes is 10, chromosomes have been generated. Otherwise go back to step 3.



index

Index	Brand	Cluster
1301	KYOKUYO Co., Ltd	A
1305	Daiwa ETF-TOPIX	B
1306	TOPIX Exchange Traded Fund	A
1308	Listed Index Fund TOPIX	C
1309	SSE50 Index Linked Exchange Traded Fund	A
1310	Daiwa ETF TOPIX Core30	B
1311	TOPIX Core 30 Exchange Traded	A
1312	Russell/Nomura Small Cap Core Index Linked ETF	C
1313	SAMSUNG KODEX200 SECURITIES EXCHANGE ...	E
1314	Listed Index Fund S&P Japan Emerging Equity 100	D
1316	Listed Index Fund TOPIX100 Japan Large Cap Equity	A
1317	Listed Index Fund TOPIX Mid400 Japan Mid Cap Equity	A
1318	Listed Index Fund TOPIX Small Japan Small Cap Equity	C
1319	Nikkei 300 Stock Index Listed Fund	A
1320	Daiwa ETF-Nikkei 225	B
1321	Nikkei 225 Exchange Traded Fund	C
1322	Listed Index Fund China A Share (Panda) CSI300	E
1323	NEXT FUNDS FTSE/JSE Africa Top40 Linked Exchange ...	B
1324	NEXT FUNDS Russia RTS Linked Exchange Traded Fund	A

Figure 8. Correspondence table of stock's index and affiliated cluster

TABLE II. INVESTOR'S ATTITUDE TOWARDS RISK CLASSIFICATION

Risk Averse type	Weak risk Averse type	Risk Neutral type	Weak risk Seeking type	Risk Seeking type
A	B	C	D	E

C. *Evaluations of the Investors*

Evaluation of investor's preference is obtained by the sum of fuzzy satisfaction of risk, return, and budget. Each satisfaction level changes the shape of the membership function according to the value of α (Fig. 9). α indicates the attitude of investors toward risk.

Portfolio satisfaction is the sum of all satisfactions, and maximization of this satisfaction is the final objective of the solution.

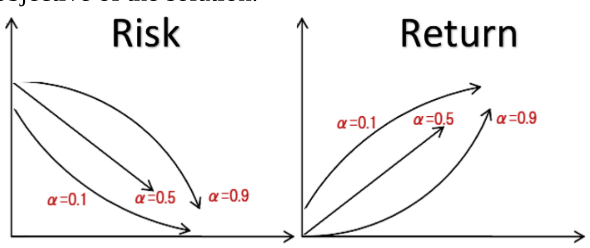


Figure 9. An example of the shape of a membership function in relation with α

An investor always has satisfaction levels for expected return and risk. In the real world of financial management, expert's knowledge and experience are very important in decision-making. Based on experts' knowledge, the investor may decide his/her satisfaction levels for expected portfolio return and risk. Watada (1997) employed a logistic function, i.e., a non-linear S shape membership function to express satisfaction levels of an investor's expected return rate and risk [7]. The S shape membership function is shown in the figure above.

$$\mu(V(x)) = \frac{1}{1+exp(\alpha(V(x)-V_M))} \tag{3}$$

$\mu(V(x))$: Risk satisfaction level

α : Investors' attitude to risk,

V_M: Risk value at where the level of membership to target risk is 0.5

$$\mu(E(x)) = \frac{1}{1+exp(-\alpha(E(x)-E_M))} \tag{4}$$

$\mu(E(x))$: Return Satisfaction level [8][14]

α : Investors' attitude to risk,

E_M : Expected return value where the level of membership to target expected return is 0.5

$$\mu(x) = e^{-b(x-K)^2}, b \geq 1 \tag{5}$$

μ (x): Budget satisfaction level

K: Budget value at which the membership level is 1.0, b: real number

Portfolio satisfaction level (Evaluation function):

$$\mu(V(x)) + \mu(E(x)) + \mu(x) \rightarrow max \tag{6}$$

D. Interface toward Investors

The selection that is operated by the investor, has 2 phases. Phase I is a cluster selection interface that selects investors' attitudes toward risks as shown in Fig. 10 (before selecting), Fig. 12 (selection for risk-averse) and Fig .14 (selection for risk-seeking). Since only the investment targets that exist in the Pareto frontier are portfolio components, investment subjects that are inferior are grayed out and displayed. Investors select three priorities from among five color-coded clusters. Investigate the rough investor's attitude to risk here by the selected cluster. Also, based on the priority determined at the time of selecting this cluster, we decide the extraction ratio of investment plan from each cluster which constitute the phase II portfolio (Table 3).

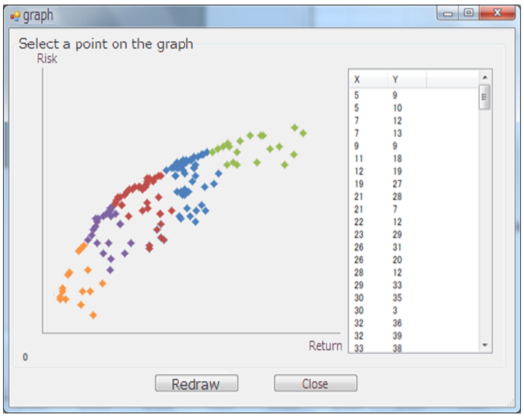


Figure 10: Interface of the Cluster selection (Phase I)

TABLE III. CLUSTER SELECTION RATIO

	First cluster	Second cluster	Third cluster
Extraction ratio	6	3	1

Phase II is the portfolio selection as shown in Fig. 11 (before selection) and Fig. 13 (after selection). Based on the ratio of the priority determined in Phase I, by extracting 10 investment targets from the 3 selected clusters, and referring to the cluster belonging from the index shown in Fig. 8, 10 portfolios will be constructed.

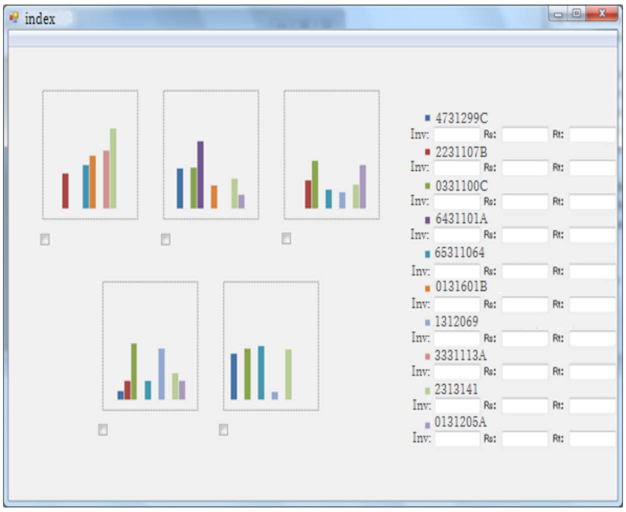


Figure 11: Interface of the Portfolio selection (Phase II)

VI. APPLICATION

A. Environments

We will demonstrate a case where the solution constitutes the optimal portfolio for investors. The return and risk values are used as actual data from a deal with Morningstar. Assuming investor's attitude is risk averse type or risk seeking type, we made two rounds of total processing. Below we will describe the investment targets that are selected for the portfolio final plan. In both cases, the investor's budget is set to 10,000 dollars.

B. Results

The results of constructing the best portfolio in the case where the attitudes of investors toward risks are assumed to be risk averse type or risk seeking type, will be illustrated below in Table 4 and 5. Risk-averse selected clusters are set to A, B, and D in descending order of priority, and risk seeking type clusters are set to E, D, B in descending order of priority. With the assumption that investors are risk averse type or risk seeking type, we will show the application of the proposed solution to the portfolio problem, the selection of the best portfolio, and the GUI created in this research along with the results.

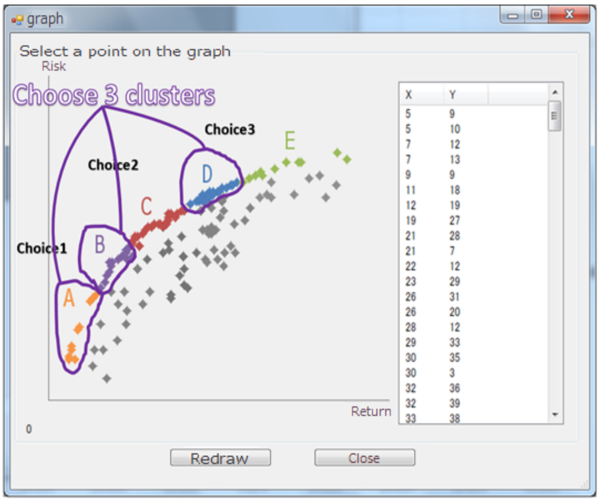


Figure 12. Selection of clusters for risk-averse investors

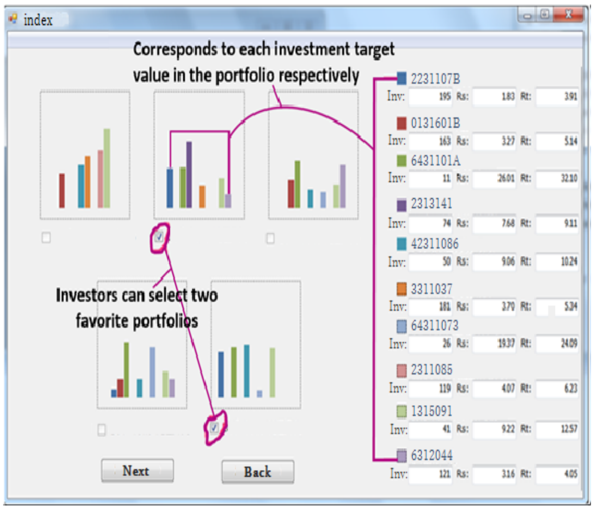


Figure 13. Selection of portfolio for risk averse-type investor

TABLE IV. RISK AVERSE-TYPE PORTFOLIO

Index	11	90	205	818	620	300	502	43	27	433
Investment amount (in 10,000)	195	163	11	74	50	181	26	119	41	121
Cluster	A	A	D	B	B	A	D	A	B	A

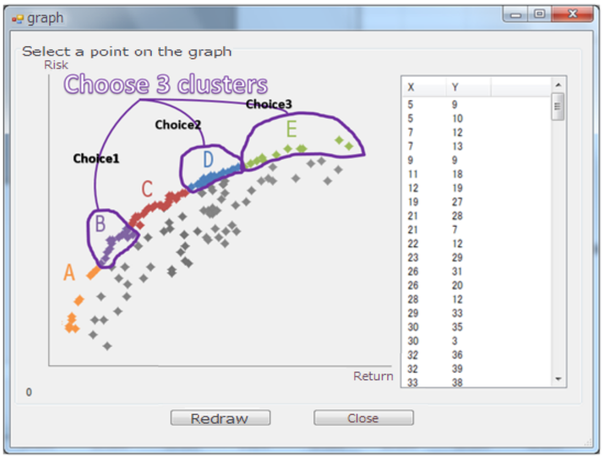


Figure 14.Selection of clusters for risk-seeking investors

TABLE V. RISK-SEEKING TYPE PORTFOLIO

Index	219	304	18	729	401	621	179	910	40	155
Investment amount (in 10,000)	169	409	76	383	85	140	262	44	102	214
Cluster	D	E	D	E	B	E	E	B	D	E

C. Discussions

The shape of the membership function expressing the risk satisfaction level of risk-averse type investors takes the shape of $\alpha = 0.1$, and is shown in the left part of Fig. 9, and the shape of the membership function representing the return satisfaction level has a shape of $\alpha = 0.9$ shown in the right part of Fig. 9, while the risk seeking type each takes the opposite shape.

This result shows that each investor's subjective preference toward risk is reflected in the membership function, and all the investor's risk, return, and budget satisfaction level of the portfolio are fully satisfied by the proposed solutions that we get in Table 4 and 5. It also shows risk-averse investors are diversifying investments to avoid risk.

In addition, because risk-averse investors are likely to have a tendency to invest in diversified portfolio to avoid risks, if the cluster selection ratio shown in Table 3 is determined in a balanced manner, it is expected that results close to preferences of existing risk-averse investors will be obtained.

And since risk-seeking type investors are likely to have a strong tendency to concentrate and invest in order to obtain returns regardless of the size of the risk, the cluster selection ratio is biased toward one with high priority once determined, we expect to have a portfolio of preferences that closely resembles existing risk-seeking investors.

VII. CONCLUSIONS

In this paper, we proposed a method for supporting investor's judgement based on Portfolio optimization using iGA that present the best compromised solution set that satisfy Pareto optimality and investor's subjective preference structures toward risk and/or return such as "risk seeking", "risk averse" and "risk neutral" by applying the interactive phase toward investors. Also, to reduce the investors' burden in this interactive phase, the cluster analysis method was proposed to narrow down the solution space in Pareto frontier which satisfy the investors' preference structure. To apply iGA for portfolio optimization problems, we proposed a two-layered chromosome representation. The first layer represents index set that figure candidacy brands of investments. The second layer represents the amount of investment for each brand which satisfy the total budget of investment.

Further, we developed an interactive system that present investment plan suitable for the investors based on his/her preference structure and show the effectiveness of the system by evaluating the proposed investment plan using a Fuzzy Portfolio model, by utilizing a membership function that can reflect the investor's satisfaction toward the proposed solution.

REFERENCES

[1] J. Lu, D. Ruan, and G. Zhang, *E-Service Intelligence: Methodologies, Technologies and Applications Journal of Economic Perspectives*, Germany: Springer, 2007, pp. 457.

[2] D. W. Gong and F. P. Pan, *Theory and Applications of Adaptive Genetic Algorithms*, Xuzhou, China: China University of Mining and Technology Press, 2003. (in Chinese)

[3] R. P. Wiegand, "An analysis of cooperative coevolutionary algorithms," Ph.D. Dissertation, George Mason University, Fairfax, 2003.

[4] N. Tokui and H. Iba, "Music composition with interactive evolutionary computation," in *Proc. the 3rd International Conference on Generative Art, Milan*, Italy, pp. 215–226, 2000.

[5] K. Deb, A. Pratap, and S. Agarwal, "A fast and elitist multiobjective genetic algorithm NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[6] H. Takagi and M. Ohsaki, "Interactive evolutionary computationbased hearing aid fitting," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 3, pp. 414–427, 2007.

[7] J. Watada, "Fuzzy portfolio selection and its applications to decision making," *Tatra Mountains Mathematical Publications*, vol. 13, pp. 219–248, 1997.

[8] http://www.investopedia.com/terms/p/portfoliomanagement.asp, Retrieved May 2017.

[9] H. M. Markowitz, "Portfolio selection," *The Journal of Finance*, vol. 7, no. 1, pp. 77-91, 1952.

[10] K. Miettinen, *Nonlinear Multiobjective Optimization*, Springer, 1999, ISBN 978-0-7923-8278-2.

[11] C. Hwang and A. S. Md Masud (1979), *Multiple Objective Decision Making, Methods and Applications: A State-of-the-art Survey*, Springer-Verlag, 1979.

[12] J. A. Hartigan and M. A. Wong, "Algorithm AS 136: A K-means clustering algorithm," *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, vol. 28, no. 1, pp. 100–108, 1979.

[13] M. Melanie, *An Introduction to Genetic Algorithms, Cambridge*, MA: MIT Press, 1996.

[14] R. Poli, W. B. Langdon, and N. F. McPhee, *A Field Guide to Genetic Programming*, 2008.

[15] M. Sasaki, "Interactive genetic algorithm for nurse-scheduling problems," *Inter. Conf. on IML*, 2011.

[16] A. M. Brintrup, J. Ramsden, H. Takagi, and A. Tiwari, "Ergonomic chair design by fusing qualitative and quantitative criteria using interactive genetic algorithm," *IEEE Trans. on Evolutionary Computation*, vol. 12, no. 3, pp. 343-354, 2008.

[17] M. Sasaki, M. Yamashiro, and A. Laamrani, "Portfolio optimization by interactive genetic algorithm," in *Proc. 22th Asia Pacific DSI Conference*, pp. 9-25, July 2017.

Masato Sasaki is an Associate Professor in the Division of System and Information Engineering at Ashikaga Institute of Technology, Japan. He received his Ph.D. from Hokkaido University and M.E. from Tsukuba University, Japan. His research interests include Multiple Criteria Decision Making, Optimization and Intelligent Systems. He has published 10 books and over 20 peer reviewed papers.

Anas Laamrani is a student of Master of Engineering, Information Science and Manufacturing Engineering, specialized in Systems and Information Engineering, Ashikaga Institute of Technology. He's a computer science engineer who graduated from Ensias, Rabat, his research interests include Optimization of Industrial Problems.

Matsuo Yamashiro is a Professor in the Division of System and Information Engineering at Ashikaga Institute of Technology, Japan. He received his Ph.D. from Keio University, Japan. His research interests include Analysis of network by GERT, Analysis of production systems by Petri-net.

Kamoyedji Ariel is a student of Master of Engineering, Information Science and Manufacturing Engineering, specialized in Systems and Information Engineering, Ashikaga Institute of Technology. He graduated from African Univ. of Technology & Management, Benin.

Tsegaye Chalew Alehegn is a student of Master of Engineering, Information Science and Manufacturing Engineering, specialized in Systems and Information Engineering, Ashikaga Institute of Technology. He graduated from Mekelle University BSC in Industrial Engineering, Ethiopia.