A Discriminant Analysis and Goal Programming Approach to solve the Multiple Criteria Data Envelopment Analysis Model

Kim F Lam
Department of Management Sciences, City University of Hong Kong, Kowloon, Hong Kong
Email: msblam@cityu.edu.hk

Abstract— **Data Envelopment Analysis (DEA) is a method measuring the relative performance of a group of decision making units (DMUs) which consume a number of inputs and produce several outputs at different quantities. In spite of its popularity, DEA still endures some kinds of shortcomings. For instance, DEA lacks the discriminating power among efficient DMUs. In this paper, we introduce a method which utilizes goal programming and discriminant analysis to solve the multiple criteria DEA model. The proposed method develops a classification function which separates efficient and inefficient DMUs and generates an efficiency ranking for all DMUs. Furthermore, it allows decision-makers to incorporate a priori information about the factor weights via proportional virtual weights restrictions or other forms of weights restrictions. Performance of the proposed method is illustrated by two real applications, which have been studied in the literature.**

Index Terms— **common set of weights, data envelopment analysis, discriminant analysis, integer linear programming, multiple criteria data envelopment analysis**

I. Introduction

With n decision making units, DMU$_j$ ($j = 1, 2, \ldots, n$), and each has m inputs x_{ij} ($i = 1, \ldots, m$) and s outputs y_{rj} ($r = 1, \ldots, s$), the CCR model [1] can be stated as follows:

$$Max \sum_{r=1}^{s} u_r y_{ro} \tag{1}$$

$$\text{s.t.} \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, j = 1,\ldots,n, \tag{2}$$

$$\sum_{i=1}^{m} v_i\, x_{io} = 1, \tag{3}$$

where $u_r, v_i \geq 0$, $r = 1, \ldots, s$; $i = 1, \ldots, m$.

A common problem in DEA is that all efficient DMUs have the same efficiency ratio; therefore, they are not fully ranked. Some studies [2-10] propose different approaches to provide further discrimination among efficient DMUs.

Manuscript received July 1, 2018; revised October 5, 2018.

A recent model proposed by Lam [11] uses discriminant analysis (DA) to rank DMUs in DEA. Recently, multiple criteria DEA analysis models have been received some attentions in the literature. Reference [12] applies multi-objective linear programming approach to solve the resource allocation problem in DEA. Reference [13] examines problem of inconsistency in several existing goal programming multiple criteria DEA models. Reference [14] applies multiple criteria sorting methods based on DEA and also Interval Analytic Hierarchy Process to evaluate research and development projects. Reference [15] suggests using a larger value for the lower bound on weights to improve discriminating power in multiple criteria DEA models.

This paper proposes a new goal programming (GP) and DA model to rank DMUs in DEA using common weights. The new model has a few advantages over the existing models: (i) it provides the minimum misclassification solution when separating efficient and inefficient DMUs in DEA using common weights, (ii) it allows decision-makers to incorporate their preferences or a priori information using virtual weights restrictions or other forms of weights restrictions, and (iii) it provides an efficiency ranking for all DMUs in a DEA study. A new GP and DA model is introduced in Section II. In Section III, the new model is applied to two applications, which have been studied in the literature. Finally, a conclusion is given in Section IV.

II. A New Goal Programming and Discriminant Analysis Model

A. Model Formulation

In this paper, we propose using GP and DA to enhance the discriminating power in DEA. The proposed method requires two steps. In the first step, the CCR model is used to determine the efficiency of each DMU. Then based on their efficiencies, DMUs are classified as either efficient (E) or inefficient ($\overline{E}$). In the second step, decision-makers incorporate a priori information using weights restrictions in the model. The model then develops a discriminant function separating efficient and inefficient DMUs. Normalized scores are used in the model. After normalization, the mean score of each input

doi: 10.18178/joams.6.4.222-226

and the mean score of each output are equal to one. Once the discriminant function is developed, it can be used to preserve an efficiency ranking of all DMUs.

Our proposed preemptive mixed integer linear goal programming and discriminant analysis model (GPDA) for DEA is stated as follows:

$$\text{Min } P_1\left(\sum_{j=1}^{n} z_j\right) + P_2(b) + P_3(d) + P_4(h) \tag{4}$$

$$\text{s.t. } \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + M z_j \geq 0, j \in E, \tag{5}$$

$$\sum_{r=1}^{s} u_{rj} - \sum_{i=1}^{m} v_i x_{ij} - M z_j \leq -\varepsilon, j \in \overline{E}, \tag{6}$$

$$\sum_{i=1}^{m} v_i = 1, \tag{7}$$

$$(1-\alpha_k)u_k - \sum_{r=1, r\neq k}^{s} \alpha_k u_r + b \geq 0, k = 1,\dots,s, \tag{8}$$

$$v_l + d \geq \beta_l, l = 1,\dots,m, \tag{9}$$

$$u_k - \gamma u_p + h \geq 0, \tag{10}$$

where ε is a very small positive number; M is a very large positive number; $0 \leq \alpha_k < 1,\ k = 1,\dots,s;\ 0 \leq \beta_l < 1,\ l = 1,\dots,m;\ z_j \in \{0,1\}, j = 1,\dots,n;\ b \geq 0; d \geq 0; h \geq 0; \gamma \geq 0; u_r \geq 0, r = 1, \dots, s; v_i \geq 0, i = 1, \dots, m$.

B. Model Discussions

In (5), for any DMU_j in E, if the efficiency ratio of DMU_j is less than one, or $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} < 0$, then the value of z_j must equal one. Similarly, in (6), for any DMU_j in $\overline{E}$, if the weighted sum of output is not less than the weighted sum of input by a magnitude of ε, then the value of z_j must equal one. In other words, an efficient DMU must satisfy $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geq 0$; otherwise, it will be counted as a misclassification in GPDA. Notice that, the condition $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geq 0$, can be rewritten as $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geq 1$, for $\sum_{i=1}^{m} v_i x_{ij} > 0$. Similarly, an inefficient DMU should satisfy $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} < 1$, otherwise, it will be counted as a misclassification in GPDA. This classification scheme is similar to classical DEA except that efficiency ratios are allowed to be greater than one. The super efficiency model [16] also allows the efficiency ratio of an efficient DMU under evaluation to be greater than one. However, a main difference is, our proposed model allows all efficient DMUs to have their efficiency ratios greater than one instead of allowing only one efficient DMU to have this flexibility as in the super efficiency model. In our model, any inefficient DMU, which has larger than or equal to one efficiency ratio will be classified as a misclassification. The primary goal in (4) minimizes the sum of z_j, or in other words, it minimizes the number of misclassifications according to the classification scheme E and $\overline{E}$ obtained from the CCR model in DEA.

The constraint $\sum_{i=1}^{m} v_i = 1$, is a normalization constraint. This constraint in virtual weights form can be expressed as $\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i x_{ij} = 1$. However, with normalized data the average score of input i, $(\frac{1}{n}\sum_{j=1}^{n} x_{ij})$, is equal to 1, therefore, the constraint becomes $\sum_{i=1}^{m} v_i = 1$.

The constraints $(1-\alpha_k)u_k - \sum_{r=1,r\neq k}^{s} \alpha_k u_r + b \geq 0$, $k = 1,\dots,s$, where $0 \leq \alpha_k < 1$, are derived from $\frac{u_k}{\sum_{r=1}^{s} u_r} \geq \alpha_k,\ k = 1,\dots,s$. In addition, α_k can be defined as the minimum shares of importance a decision-maker wants to maintain for output k with respect to the total output. The variable b is a deviational variable. With normalized data, the output weight u_k can be viewed as the virtual of the 'average' of output k since the mean of output k is one, therefore, $\left(\frac{1}{n}\sum_{j=1}^{n} y_{kj}\right) u_k = u_k$. Similarly the constraints, $v_l + d \geq \beta_l,\ l = 1,\dots,m$, are derived from the conditions, $\frac{v_l}{\sum_{i=1}^{m} v_i} \geq \beta_l,\ l = 1,\dots,m$, where $0 \leq \beta_l < 1$, and $\sum_{i=1}^{m} v_i = 1$.

Furthermore, β_l represents the minimum shares of importance a decision-maker wants to maintain for input l with respect to the total input. The variable d is a deviational variable. The constraint, $u_k - \gamma u_p + h \geq 0$, where $\gamma \geq 0$ and h is a deviational variable, for any pair of outputs $\{k, p\}$, is a modified version of the constraints in the assurance region analysis suggested by [17]. Similar constraint can be applied to any pair of inputs.

Notice that decision-makers do not need to include all the suggested weight restriction constraints only if the constraints are necessary. They can include one set or only a subset of each type of the suggested weight restriction constraints in GPDA. Alternatively, decision-makers can also include other types of weight restriction constraints if deemed necessary. For instance, constraints on the upper bounds of factor weights can also be included. Examples of applying weight restrictions in GPDA are illustrated in Section III .

GPDA is solved as a preemptive goal programming problem. The primary goal is to minimize the number of misclassifications of DMUs in E and $\overline{E}$. Decision-makers can rank order other goals according to their importance. The values of u_r and v_i are used to compute efficiency ratios of all DMUs. A priori available information gathered from expert opinion or preferences from decision-makers can be incorporated in GPDA. The use of weights restrictions in DEA has been discussed by many researchers. In the literature, the proposed weights restrictions are generally applied to each individual DMU under evaluation. However, in GPDA weights restrictions are applied to a common set of weights for all DMUs.

TABLE I. NORMALIZED DATA OF EXAMPLE 1

	Input			Output		
DMU	I.L.F.	W.F.	INV.	G.I.O.	P.&T.	R.S.
Beijing	2.932	3.232	4.155	3.192	3.704	3.633
Changchun	0.834	0.746	0.583	0.636	0.643	0.983
Changsha	0.482	0.404	0.478	0.344	0.274	0.753
Chengdu	1.247	1.240	0.996	0.988	0.822	1.244
Chongqing	2.059	1.806	1.210	1.395	1.034	1.486
Dalian	1.010	1.149	1.138	1.014	1.059	1.053
Fuzhou	0.426	0.393	0.594	0.299	0.240	0.631
Guangzhou	1.355	1.585	2.036	1.552	1.501	2.375
Guiyang	0.466	0.412	0.439	0.347	0.397	0.318
Hangzhou	0.913	0.865	0.689	0.976	0.868	1.166
Harbin	1.282	1.451	0.797	0.904	0.678	1.095
Hefei	0.453	0.387	0.325	0.363	0.340	0.422
Hohot	0.270	0.228	0.244	0.171	0.180	0.242
Jinan	0.702	0.619	0.449	0.594	0.594	0.693
Kunming	0.734	0.684	0.648	0.591	0.641	0.610
Lanzhou	0.955	0.958	0.616	0.781	0.766	0.611
Lhasa	0.012	0.008	0.042	0.002	0.002	0.113
Nanchang	0.620	0.525	0.245	0.426	0.335	0.472
Nanjing	1.122	1.095	1.277	1.125	1.133	1.073
Nanning	0.301	0.000	0.210	0.237	0.759	0.356
Ningbo	0.360	0.284	0.418	0.431	0.406	0.814
Shanghai	5.500	6.418	5.503	9.067	10.239	5.202
Shenyang	1.826	1.706	1.139	1.368	1.067	1.441
Shenzhen	0.091	0.144	1.596	0.105	0.062	0.789
Shijiazhuang	0.662	0.565	0.394	0.610	0.513	0.496
Taiyuan	1.020	0.899	0.980	0.691	0.462	0.539
Tianjin	2.617	2.556	3.107	3.029	2.931	1.958
Urumqi	0.425	0.385	0.750	0.275	0.209	0.376
Wuhan	1.726	1.801	1.055	1.638	1.608	1.446
Xiamen	0.172	0.190	0.560	0.176	0.205	0.315
Xian	1.229	1.198	0.808	0.855	0.552	0.935
Xining	0.280	0.302	0.262	0.135	0.101	0.244
Yinchuan	0.135	0.126	0.264	0.080	0.062	0.149
Zhengzhou	0.747	0.586	0.498	0.555	0.576	0.670
Zhuhai	0.034	0.051	0.496	0.048	0.036	0.297

III. COMPUTATIONAL EXAMPLES

A. Example 1

The data used in Example 1 was previously studied by [17-18]. Reference [17] applies DEA/assurance region analysis to measure the "allocative" efficiency of the industrial performance of 35 selected Chinese cities. Reference [18] determines a common set of weights using the proposed method, Discriminant Data Envelopment Analysis of Ratios, to rank the 35 Chinese cities. A detail description of the inputs and outputs, and their original values of the 35 Chinese cities are given by [17].

TABLE II. APPLYING GPDA* USING ASSURANCE REGION (GOAL 2 TO GOAL 5) SUGGESTED BY [17] TO EXAMPLE 1

	Constraints imposing weights restrictions in GPDA	Values of deviational variables
Goal 1: Min $\sum_{j=1}^{35} z_j$		$\sum_{j=1}^{35} z_j = 1$
Goal 2: Min d_2	$v_2 - 2v_1 + d_2 \geq 0$	$d_2 = 0$
Goal 3: Min d_3	$u_2 - 0.5u_1 - d_3 \leq 0$	$d_3 = 0$
Goal 4: Min d_4	$v_3 - 2v_1 + d_4 \geq 0$	$d_4 = 0$
Goal 5: Min d_5	$u_3 - 0.5u_1 - d_5 \leq 0$	$d_5 = 0$
Goal 6: Min d_6	$v_1 + d_6 \geq 0.05$ $v_2 + d_6 \geq 0.05$ $v_3 + d_6 \geq 0.05$	$d_6 = 0$
Goal 7: Min d_7	$0.95u_1 - 0.05u_2 - 0.05u_3 + d_7 \geq 0$ $0.95u_2 - 0.05u_1 - 0.05u_3 + d_7 \geq 0$ $0.95u_3 - 0.05u_1 - 0.05u_2 + d_7 \geq 0$	$d_7 = 0$

*β = 0.05 and α = 0.05

To normalize the data, each input or output score is divided by its average value. The normalized data is given in Table I. A list of applied weight restriction constraints are provided in Table II. Notice that, constraints of Goal 2 to Goal 5 are adapted from the weights restrictions of an assurance region suggested by [17]. Constraints of Goal 6 and Goal 7 are obtained by setting $\beta = \alpha = 0.05$. We decide to use smaller values of β and α in Goal 6 and Goal 7.

After the above constraints and parameters have been set, GPDA is solved by preemptive goal programming. First, GPDA is solved with only the primary goal of minimizing the total number of misclassifications. The solution indicates that one city is being misclassified. Then, adding the constraints: $\sum_{j=1}^{35} z_j = 1$ and the constraint of Goal 2, $(v_2 - 2v_1 + d_2 \geq 0)$, GPDA is solved again, and this time the objective is Min d_2. Applying similar procedures, the values of all deviational variables are reported in Table II.

The factor weights obtained from GPDA are reported in Table III and rankings of cities of different methods are reported in Table IV. In Table IV, [17] misclassified four cities (i.e. either {Lhasa, Zhuhai, Shijiazhuang, Wuhun} or {Lhasa, Zhuhai, Hangzhou, Nanning}) while [18] misclassified two cities (i.e. Hangzhou and Nanchang). GPDA misclassified only one city (i.e. Nanchang). Therefore, GPDA has the best performance in terms of the accuracy of classifying efficient and inefficient DMUs in this study.

TABLE III. WEIGHTS OBTAINED FROM GPDA IN EXAMPLE 1

v_1	v_2	v_3	u_1	u_2	u_3
0.05000	0.82949	0.12051	0.51954	0.04102	0.25977

TABLE IV. RANKINGS OF CITIES VIA DIFFERENT METHODS

Cities	Efficient Cities	[17]	[18]	GPDA
Beijing		16	7	10
Changchun		18	15	9
Changsha		27	19	8
Chengdu		25	27	23
Chongqing		13	24	29
Dalian		23	13	21
Fuzhou		30	23	18
Guangzhou		19	8	7
Guiyang		26	21	26
Hangzhou	yes	6	**10***	5
Harbin		17	31	31
Hefei		14	20	13
Hohot		29	25	24
Jinan		9	14	11
Kunming		24	18	22
Lanzhou		10	26	28
Lhasa	yes	**35***	1	2
Nanchang	yes	1.5	**29***	**25***
Nanjing		12	12	14
Nanning	yes	7	2	1
Ningbo	yes	1.5	3	3
Shanghai	yes	3	4	4
Shenyang		11	28	27
Shenzhen		31	6	19
Shijiazhuang		**4***	22	15
Taiyuan		28	34	34
Tianjin		8	11	16
Urumqi		32	32	33
Wuhan		**5***	16	20
Xiamen		20	9	17
Xian		21	33	30
Xining		34	35	35
Yinchuan		33	30	32
Zhengzhou		15	17	12
Zhuhai	yes	**22***	5	6

*Misclassified

B. *Example 2*

The data used in Example 2 was previously studied by [4-5, 19]. The original data can be found in [19]. It contains three inputs and three outputs of 20 bank branches located in a region in Iran. The normalized data is given in Table V. Suppose decision makers in general agree that all inputs and outputs are significantly important in determining efficiencies of DMUs, then decision makers may want to set $\beta > 0$ and $\alpha > 0$ in GPDA. By setting $\beta = \alpha = 0.2$, the goals and the weight restrictions constraints of GPDA are given in Table VI. After we solved GPDA, the values of all the deviational variables are also reported in Table VI. In Example 2, all three goals are maintained and all deviational variables equal zero. The factor weights obtained from GPDA are reported in Table VII. Rankings from different methods are reported in Table VIII. In Table VIII, all efficient bank branches have higher rankings than inefficient bank branches when evaluated by [5] and GPDA. The model proposed by [4] ranks only efficient bank branches. In this example, GPDA has similar performance when compare with other existing methods.

TABLE V. NORMALIZED DATA OF EXAMPLE 2

	Inputs			Outputs		
Bank Branch	Staff	Computer Terminals	Space (m^2)	Deposits	Loans	Charge
1	1.2871	0.9825	0.4216	0.9963	0.9502	0.7974
2	1.0784	0.8421	2.7202	1.1882	1.1433	1.2602
3	1.0811	1.0526	1.3941	1.1971	1.7682	0.7102
4	1.1717	0.7719	0.5712	1.0105	1.1524	2.7253
5	1.1039	1.1930	0.7276	1.2234	1.3159	0.6712
6	1.1398	0.9123	1.3601	1.0849	1.0979	1.5504
7	0.9737	0.8421	0.9521	0.9565	1.6401	1.9508
8	1.0636	1.0526	0.3264	0.6555	0.4264	0.8113
9	0.6441	0.8421	0.3672	0.4200	0.6639	0.6647
10	0.9185	0.7719	1.3873	0.4289	0.3344	0.1325
11	0.9632	1.4035	0.8297	1.1101	0.5793	1.0986
12	1.0988	0.9123	0.6936	0.6434	1.6811	1.7112
13	0.8920	1.1930	0.9249	0.9203	1.1758	0.7099
14	1.3223	1.1228	1.4689	0.7567	0.9372	0.6631
15	0.9271	1.3333	1.2241	5.2437	0.4769	0.2676
16	0.8298	1.2632	1.4281	0.6036	0.7328	1.2648
17	1.3544	0.8421	0.5576	0.4719	1.8223	0.4399
18	0.8583	0.9123	0.6392	0.3099	0.6364	0.1848
19	0.5031	0.9825	0.6460	0.2019	0.3459	0.3031
20	0.7892	0.7719	1.3601	0.5773	1.1198	2.0830

TABLE VI. APPLYING GPDA* TO EXAMPLE 2

Preemptive goals	Constraints imposing weights restrictions in GPDA	Values of deviational variables
Goal 1: Min $\sum_{j=1}^{20} z_j$		$\sum_{j=1}^{20} z_j = 0$
Goal 2: Min d_2	$v_1 + d_2 \geq 0.2$ $v_2 + d_2 \geq 0.2$ $v_3 + d_2 \geq 0.2$	$d_2 = 0$
Goal 3: Min d_3	$0.8u_1 - 0.2u_2 - 0.2u_3 + d_3 \geq 0$ $0.8u_2 - 0.2u_1 - 0.2u_3 + d_3 \geq 0$ $0.8u_3 - 0.2u_1 - 0.2u_2 + d_3 \geq 0$	$d_3 = 0$

* $\beta = 0.2$ and $\alpha = 0.2$

TABLE VII. INPUT AND OUTPUT WEIGHTS IN EXAMPLE 2

v_1	v_2	v_3	u_1	u_2	u_3
0.29131	0.26321	0.44549	0.18292	0.36427	0.36742

TABLE VIII. RANKINGS OF BANK BRANCHES VIA DIFFERENT METHODS

Branch	CCR-efficient	[4]	[5]– DMU_1	[5] – DMU_4	GPDA
1	yes	7	6	5	7
2			10	10	16
3			8	8	11
4	yes	2	1	3	1
5			9	9	9
6			14	14	8
7	yes	3	2	1	2
8			12	12	14
9			13	13	10
10			20	20	20
11			16	16	13
12	yes	6	4	6	3
13			11	11	12
14			18	18	17
15	yes	1	7	2	5
16			15	15	15
17	yes	5	5	7	6
18			17	17	18
19			19	19	19
20	yes	4	3	4	4

IV. CONCLUSION

One of the common criticisms in DEA is that it lacks discriminating power among efficient DMUs, since they all have the same efficiency ratio. In this paper, we introduce a method, GPDA, based on discriminant analysis and weight restrictions to enhance the discrimination power in DEA. The advantages of applying GPDA have already been discussed in the paper. The results of the two empirical examples also demonstrate the usefulness of applying GPDA to rank DMUs. Future research areas may include adding more forms of weight restrictions to GPDA. Currently, GPDA applies symmetric penalty to both the misclassifications of an efficient DMU and an inefficient DMU. However, it may be more appropriate to put different penalties on the misclassification of inefficient DMUs and efficient DMUs depending on their expected costs of misclassifications.

REFERENCES

[1] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of the decision making units," *European Journal of Operational Research*, vol. 2, pp. 429-444, 1978.

[2] T. R. Sexton, R. H. Silkman, and A. J. Hogan, "Data envelopment analysis: critique and extensions," in *Measuring Efficiency: An Assessment of Data Envelopment Analysis*, R. H. Silkman, Ed. San Francisco, CA: Jossey-Bass, 1986, pp. 73-105.

[3] J. Doyle and R. Green, "Efficiency and cross-efficiency in DEA: derivations, meanings and uses," *Journal of the Operational Research Society*, vol. 45, pp. 567-578, 1994.

[4] G. R. Jahanshahloo, H. V. Junior, F. H. Lotfi, and D. Akbarian, "A new DEA ranking system based on changing the reference set," *European Journal of Operational Research*, vol. 181, pp. 331-337, 2007.

[5] G. R. Jahanshahloo, F. H. Lotfi, M. Khanmohammadi, M. Kazemimanesh, and V. Razaie, "Ranking of units by positive ideal DMU with common weights," *Expert Systems with Applications*, vol. 37, pp. 7483-7488, 2010.

[6] K. F. Lam, "In the determination of weight sets to compute cross-efficiency ratios in DEA," *Journal of Operational Research Society*, vol. 61, pp. 134-143, 2010.

[7] K. F. Lam and F. Bai, "Minimizing deviations of input and output weights from their means in data envelopment analysis," *Computers & Industrial Engineering*, vol. 60, pp. 527-533, 2011.

[8] S. Lim, "Minimax and Maximin formulations of cross-efficiency in DEA," *Computers & Industrial Engineering*, vol. 62, pp. 726-731, 2012.

[9] K. F. Lam, "In the determination of the most efficient decision making unit in data envelopment analysis," *Computers & Industrial Engineering*, vol. 79, pp. 76-84, 2015.

[10] A. Oukil and G. R. Amin, "Maximum appreciative cross-efficiency in DEA: A new ranking method," *Computers & Industrial Engineering*, vol. 81, pp. 14-21, 2015.

[11] K. F. Lam, "Finding a common set of weights for ranking decision-making units in data envelopment analysis," *Journal of Economics, Business and Management*, vol. 4, pp. 534-537, 2016.

[12] J. Wu, Q. Zhu, Q. An, J. Chu, and X. Ji, "Resource allocation based on context-dependent data envelopment analysis and a multi-objective linear programming approach," *Computers & Industrial Engineering*, vol. 101, pp. 81-90, 2016.

[13] A. P. D. Santos Rubem, J. C. C. B. Soares de Mello, and L. Angulo Meza, "A goal programming approach to solve the multiple criteria DEA model," *European Journal of Operational Research*, vol. 260, pp. 134-139, 2017.

[14] E. Karasakal and P. Aker, "A multicriteria sorting approach based on data envelopment analysis for R&D project selection problem," *Omega*, vol. 73, pp. 79-92. 2017.

[15] A. Hatami-Marbini and M. Toloo, "An extended multiple criteria data envelopment analysis model," *Expert Systems with Applications*, vol. 73, pp. 201-219, 2017.

[16] P. Andersen and N. Petersen, "A procedure for ranking efficient units in data envelopment analysis," *Management Science*, vol. 39, pp. 1261-1264, 1993.

[17] T. Sueyoshi, "Measuring the industrial-performance of Chinese cities by Data Envelopment Analysis," *Socio-Economic Planning Sciences*, vol. 2, pp. 75-88, 1992.

[18] L. Friedman and Z. Sinuany-Stern, "Combining ranking scales and selecting variables in the DEA context: the case of industrial branches," *Computers & Operations Research*, vol. 25, pp. 781-791, 1998.

[19] A. Amirteimoori, and S. Kordrostami, "Efficient surfaces and an efficiency index in DEA: a constant returns to scale," *Applied Mathematics and Computation*, vol. 163, pp. 683-691, 2005.

Kim F Lam received a BBA (1984), an MBA (1986) and a PhD (1992) in management science at Simon Fraser University, Vancouver, B.C., Canada.

He is an Associate Professor of Management Science at City University of Hong Kong, Hong Kong. He has published in journals such as *Annals of Operations Research, Computers and Industrial Engineering, Computers and Operations Research, European Journal of Operational Research, Journal of the Operational Research Society,* and *Personnel Review*, etc. His research interests include multi-criteria decision making, discriminant analysis, and data envelopment analysis.

Dr. Lam is a member of the International Society on Multiple Criteria Decision Making.