Abstract—This paper uses Bayesian model to propose two different conditions in which the ordering quantity and ordering time are two previous and relative data. Based on cost balance view of point, productivity is the prime goal in material manufacturing process. Thus, this research calculates the full model of productivity with considering these two different conditions cases. The empirical data is used to estimate parameters of these two proposed models and make the comparisons of model calibration between these two conditions. The results show good fitness between empirical data and simulation data when ordering time is conditioned by ordering quantity. Finally the conclusion is made for practical application in operations and production management.

Index Terms—material demand, bayesian model, ordering quantity, ordering time, productivity, cost balance

I. INTRODUCTION

To forecasting material demand in manufacturing process, the ordering quantity and ordering time are two variables which are most used to construct the demand model. There are various relationships of these two variables are considered such as considering that ordering quantity follows the lognormal distribution and the factor of ordering time is the recency time which follows exponential distribution and demonstrated as renew process[3]. The recency of ordering time reveals the clue that upstream manufactures may gradually run out or use up their inventory of materials if the recency of ordering time is longer. Some researches[5,6] consider Farlie-Gumbel-Morgenstern family of bivariate distributions to portray the ordering quantity from different sources. Some researches [1] [4,5,6] focus on the material demand quantity from two different sources of downstream. The extended model [4] considers the ordering quantity from different sources and uses the characteristic function to describe calculate the probability density function of total ordering quantity. It demonstrates not only the information of ordering quantity of past in downstream manufacturer itself but also reflect the various ordering input from upstream.

On the other hand, Huang [6] stands on the view of point of manufacture’s productivity to propose a model which can help manager to control their cost and make a balance when ordering their materials. The model development of cost release a general function which makes it possible to extend to different distributions depending on different kind of cost or different type of industries.

Thus this paper bases on the concept of productivity, in order to achieve a good productivity, the cost is considered into the proposed model when predicting the demand in the producing process. This paper also uses ordering quantity and ordering time as previous data with conditional probability.

This article is organized as following: first the proposed model is present. There are two condition of portraying different kinds of impact between ordering quantities and ordering time. Some probability densities are demonstrated to construct the model. Secondly, the parameters of this model are estimated by empirical data collection. Thirdly, the comparison between empirical data and simulation data are calculated for model calibration. Finally, the conclusions are made.

II. THE MATERIAL DEMAND MODE

According to Huang [7], ordering quantity and ordering time are considered as before and after relationship. In condition 1, it demonstrates ordering time is conditioned by ordering quantity which means managers decide how many material quantities they will order before getting the information of ordering time. In the model, other exogenous variable are considered through linear equation. In condition 2, it presents the ordering quantity is conditioned by ordering time which means the decision of ordering material happens after ordering time. Finally, the empirical data is conduct to make parameters estimation and model calibration. And the conclusions are made.

A. Condition 1: To Observe Ordering Quantity, Then Predicting Ordering Time

Based on Huang[7]'s assumption, in condition 1, first, to observe the ordering quantity from downstream then using this information to decide the ordering time. The ordering quantity q is considered as a random variable and follows normal distribution with parameter τ and ε². τ> 0. The ordering time s is also a random variable which is conditioned by q follows exponential with parameter δ. We consider Bayesian model. Let δ is a random variable with its prior density as \( f(\delta; \tau, \varepsilon^2) = \frac{\delta^{-\tau - \varepsilon^2} e^{-\delta}}{\Gamma(\tau)} \). Then its
marginal density of $\delta$ is $l_\delta(s; \tau, e^2) = \frac{\tau e^{2\tau}}{(s + e^2)^{\tau+1}}$. Then the joint density of ordering quantity $q$ and ordering time $s$ can be calculated by

$$h_1(q, s) = \frac{\tau e^{2\tau}}{2\pi e^2(s + e^2)^{\tau+1}} \exp \left[\frac{-(q - \tau)^2}{2e^2}\right]. \quad (1)$$

B. Full Model in Condition 1

Based on inventory theory, the productivity is the output of the material manufacturing process divided by the input. The output is defined as the material demand providing. The input is focused on the cost which is from material producing. And Huang\cite{6} considers that the cost is composed from inventory and material shortage. The extended Farlie-Gumbel-Morgenstern family of bivariate distributions is composed from inventory and material shortage. The material producing. And Huang\cite{6} considers that the cost is included inventory cost (a random variable $m$) and shortage cost (a random variable $g$) as bivariate distributions used to portray these two different cost. Then we can obtain the cumulative distribution function(c.d.f.) of productivity as

$$F(p) = \int_0^p P\left(\frac{w}{c} < p|\text{output} = w\right) \cdot f(w) dw$$

$$= \int_0^p \frac{w}{c} < p \cdot f(w) dw$$

$$= \int_0^p \frac{c > w}{p} \cdot f(w) dw \quad .\quad (2)$$

In this equation, $w$ is denoted as the material demand. In the conditional 1, we use $h_1(q, s) = \frac{\tau e^{2\tau}}{2\pi e^2(s + e^2)^{\tau+1}} \exp \left[\frac{-(q - \tau)^2}{2e^2}\right]$ to replace $f(w)$.

We also consider the cost which is included inventory cost (a random variable $m$) and shortage cost (a random variable $g$) as bivariate distributions in which there are three dimensional parameters, $\alpha$, $\beta$ and $\gamma$.

Thus, the c.d.f. of full model in conditional 1 is

$$F_1(p) = \frac{\tau}{\sqrt{2\pi e^2}} \int_0^\infty \frac{h(m) k(g)}{\sqrt{2\pi e^2}} \cdot \exp \left[\frac{-(q - \tau)^2}{2e^2}\right] \, dm \, dg \, ds \quad .\quad (3)$$

Based on equation (3), we can calculate the probability density function(p.d.f.) of the full model as

$$f_1(p) = \frac{df_1}{dp} = \frac{\tau}{\sqrt{2\pi e^2}} \int_0^\infty \frac{h(m) k(g)}{\sqrt{2\pi e^2}} \cdot \exp \left[\frac{-(q - \tau)^2}{2e^2}\right] \, dm \, dg \, ds \quad .\quad (4)$$

C. Condition 2: To Observe Ordering Time, then Predicting Ordering Quantity

In this condition, the ordering time from downstream is first observed, then the information is collected to decide the ordering quantity. Based on Huang\cite{7}'s assumption, the random variable $o$ is denoted as ordering time which follows exponential distribution with parameter $\theta$. And the ordering quantity $z$ which is conditioned by $o$ is followed as normal distribution with parameters $r(o)$ and $\sigma^2(o)$. These two parameters are functional equations of ordering time $o$. In $r(o)$, we consider there is a linear relation with constant $a_0$, parameter $a_1$ of variable $o$ and parameter $a_2$ of variable $k$. $k$ is an explanatory variable of exogenous dynamic influence factor from material demand.

We consider $\sigma^2(o)$ is a positive value with $u$ power of $o$ in which we propose a Bayesian model as

$$g_x(u) = \frac{1}{\sqrt{2\pi \sigma^2(o)}} \exp \left[\frac{(2\sigma^2)^{-1} (ln o) (u - a_o - a_0 - a_k)^2 - o \cdot \theta}{2 \sigma^2}\right]$$

Then the joint density of ordering quantity $u$ and ordering time $o$ is demonstrated by

$$h_2(u, o) = \frac{\theta}{\sqrt{2\pi \sigma^2(o)}} \exp \left[\frac{(2\sigma^2)^{-1} (ln o) (u - a_o - a_0 - a_k)^2 - o \cdot \theta}{2 \sigma^2}\right]$$

$h_2(u, o)$ is a joint density function of ordering quantity $u$ and ordering time $o$.

D. Full Model in Condition 2

Based on the balance model\cite{6}, we can calculate the c.d.f. of full model in conditional 2. We also consider the cost which is included inventory cost (a random variable $m$) and shortage cost (a random variable $g$) as bivariate distributions in which there are three dimensional parameters. And according to equation (3), the c.d.f. of full model in conditional 2 can be calculated by

$$F_2(p) = \frac{\tau}{\sqrt{2\pi e^2}} \int_0^\infty \frac{h(m) k(g)}{\sqrt{2\pi e^2}} \cdot \exp \left[\frac{-(q - \tau)^2}{2e^2}\right] \, dm \, dg \, ds$$

Based on equation (6), we can calculate the probability density function(p.d.f.) of the full model as
Both simulation data and empirical data are used to make model calibration. The data analysis steps are following: (1) half of empirical data are used to estimate the parameters of these two cases. (2) the simulation data is got from the proposed model from condition 1 and 2. (3) we make comparison between the other half empirical data and the simulation data to find which case is better good fitness and more close to the real situation. We use Mean Absolute Percentage Error (MAPE) to calculate the difference between real (empirical) data and simulation data for model calibration.

A. Empirical Data

The empirical data is collected from January 1 to July 31 in 2017. It includes the quantities of Nano material panel ordering quantities and ordering time from an IC company. This company purchase the Nano material panel as their material demand and produce mobile phone components to its downstream manufactures.

In condition 1, based on a conditional probability of ordering quantity, we propose the model of ordering time. \( b_1, b_2 \) are the interval of ordering time. Ordering time and ordering quantity are pair data as \((b_1, j_1), (b_2, j_2)\). We discard the last data of ordering quantity \( b_1 \).

In condition 2, we make a conditional probability of ordering time to propose the model of ordering quantity. \( c_1, c_2 \) are the interval of ordering time and \( y_1, y_2 \) are the ordering quantities. Ordering time and ordering quantity are pair data as \((c_1, y_1), (c_2, y_2)\). We discard the first data of ordering quantity \( y_0 \).

B. Analysis Process

In the data analysis process:

(1) We first separate empirical data into two parts. One is for parameters estimation. Another is for model calibration.

(2) Then use the results of parameters estimation to simulate the data from the proposed model distributions.

(3) Finally, we compare the simulation data and the other parts of empirical data.

(4) To find the difference (distance) between the simulation data and empirical data.

IV. THE RESULTS

Both simulation data and empirical data are used to make model calibration. The data analysis steps are following: (1) half of empirical data are used to estimate the parameters of these two cases. (2) the simulation data is got from the proposed model from condition 1 and 2. (3) we make comparison between the other half empirical data and the simulation data to find which case

A. The Results of Parameters Estimation

| TABLE I. THE RESULTS OF PARAMETERS ESTIMATION IN CONDITION 1 AND 2 |
|-----------------|-----------------|-----------------|
|                 | Condition 1     | Condition 2     |
| \( \tau \)      | 3.575           | --              |
| \( \varepsilon \)| 0.355           | --              |
| \( \alpha \)     | -1.032          | -2.001          |
| \( \beta \)      | 0.299           | 0.865           |
| \( \gamma \)     | -0.862          | -1.230          |
| \( a_0 \)        | --              | 3.565           |
| \( a_1 \)        | --              | 0.783           |
| \( a_2 \)        | --              | -0.256          |
| \( \theta \)     | --              | 3.656           |
| \( i \)          | --              | 38.22           |
| \( k \)          | --              | 2.360           |
We use MLE (maximum-likelihood estimation) to estimate the parameters. And for application, the bivariate distributions which are indicated the inventory cost and shortage cost in full model are considered respectively as a log normal distribution and Rayleigh distribution.

The results are shown as Table I. After parameter estimation, the simulation data are got by the proposed model in condition 1 and 2.

B. The Results of Model Calibration

We calculate the value of MAPE. In condition 1, MAPE= 0.655. In condition 2, MAPE= 0.701. MAPE is the distance between real data and simulation data. Thus, more closer of these two data, the value of MAPE is more smaller. It shows that MAPE of condition 1 is smaller than of condition 2. Therefore, to first observe ordering quantity then predict ordering time (to assume the ordering time is conditioned by ordering quantity) is more close to the real production phoneme.

V. CONCLUSION

This paper combines the extended material demand model to consider the ordering quantity and ordering time are two relative variables and the concept of productivities to consider cost balance when forecasting material demand. Two conditions are proposed to describe whether to observe ordering time, then predicting ordering quantity or to observe ordering quantity first, then predicting ordering time. This paper also calculates the full model of both conditions to demonstrate the cumulative distribution function and probability density function of productivity.

The result is similar with Huang[7] which shows the model in which ordering time is conditioned by ordering quantity is more close to the real production phoneme. In the future, rather than normal distribution and exponential distribution, other probability distributions can be proposed to compare the difference real data and simulation data. There are also other distributions can be considered in balance model to calculate the productivity.

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