Choosing Suitable Weights Sets for Cross-Evaluations in DEA

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Abstract— **Data envelopment analysis (DEA) has been a popular approach in performance measure. However, alternate optimal solutions exist in most linear programming solutions of an efficient decision making unit (DMU), and reduce the effectiveness of the DEA cross efficiency evaluation method in ranking DMUs. Some methods choosing different weight sets in the alternate optimal solutions to perform cross efficiency evaluation have been studied in the literature. This paper introduces an approach to find two weight sets with opposite secondary objectives: one minimizes the number of efficient DMUs and the other maximizes the number of efficient DMUs, in the alternate optimal solutions, for cross efficiency evaluation. Both weight sets are used to compute cross efficiency evaluation in DEA. The intuition of this approach is that a "truly" efficient DMU is expected to perform well in any efficient weight sets in DEA. Therefore, a more efficient DMU is expected to have a higher evaluation than those of other less efficient DMUs when evaluated with weight sets that are different in their weight patterns. Computational results are provided to show the value of the proposed approach.**

Index Terms— **data envelopment analysis, discriminant analysis, efficiency ratio, linear programming, mixed-integer linear programming, performance measure**

I. Introduction

Data Envelopment Analysis (DEA) is a very popular method in performance measure. Consider n decision making units, DMU$_j$ ($j = 1,2,\ldots,n$) each of which has m inputs x_{ij} ($i = 1,\ldots, m$) and s outputs y_{rj} ($r = 1,\ldots, s$). The relative efficiency of each DMU$_0$ can be obtained from the following linear programming (LP) model [1], usually known as the CCR model:

$$Max \sum_{r=1}^{s} u_r y_{ro} \tag{1}$$

$$\text{s.t.} \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\ldots,n, \tag{2}$$

$$\sum_{i=1}^{m} v_i\, x_{io} = 1, \tag{3}$$

where u_r ($r = 1,\ldots, s$) and v_i ($i = 1,\ldots, m$) are factor weights for output r and input i, respectively.

Despite of its popularity, a common problem in DEA is that all efficient decision making units (DMUs) have the same efficiency ratio which makes it difficult to rank efficient DMUs. Reference [2] uses cross-efficiency ratios to provide discrimination among efficient DMUs. Cross-efficiency ratios of a DMU are obtained by using optimal weights from other DMUs to compute efficiency ratios. Then, the average values of cross-efficiency ratios can be used to rank DMUs in terms of their efficiencies. The argument of using cross-efficiency ratios is that an efficient DMU should maintain good performance under different weight patterns. As a result, it is expected that the higher the cross-efficiency ratio, the more efficient is the DMU. However, alternate optimal solutions exist in most linear programming solutions of an efficient DMU, and reduce the effectiveness of ranking DMUs using cross-efficiency ratio in DEA.

Methods choosing weight sets among the alternate optimal solutions to perform cross efficiency evaluation have been studied in the literature. References [3-4] propose several secondary objectives to look for a more suitable weight set within the alternate optimum region. Reference [5] combines discriminant analysis and super-efficiency DEA model in DEA cross efficiency evaluation. Reference [6] proposes a model that minimizes deviations of input and output weights from their means for efficient DMUs in data envelopment analysis. Reference [7] introduces a cross efficiency evaluation method based on weight-balanced data envelopment analysis model. Reference [8] proposes two formulations on the secondary goal namely: minimizes the best cross-efficiency of peer DMUs of cross-efficiency in DEA, and maximizes the worst cross-efficiency of peer DMUs of cross-efficiency in DEA. Reference [9] proposes new methods for ranking decision making units based on the dispersion of weights and Norm 1 in Data Envelopment Analysis. Reference [10] applies super-efficiency DEA model to find the most efficient decision making unit in DEA. Reference [11] introduces a new ranking method base on maximum appreciative e cross-efficiency in DEA. Reference [12] extends secondary goal models to incorporate weight selection in DEA cross-efficiency evaluation. Reference [13] suggests performing DEA cross-efficiency evaluation based on Pareto improvement.

Manuscript received May 10, 2018; revised January 8, 2019.

doi: 10.18178/joams.7.1.19-24

The idea of using cross efficiency evaluation is to evaluate DMU by different weight sets. Intuitively, adding more suitable weight sets in cross efficiency evaluation will increase the reliability of the evaluation. Therefore, in this paper for each efficient DMU, we introduce an approach to find two weight sets with opposite secondary objectives, namely: (1) minimizing the total number of efficient DMUs and (2) maximizing the total number of efficient DMUs, among the multiple optimal solutions in DEA for cross efficiency evaluation. A "truly" efficient DMU is expected to perform well in any efficient weight sets in DEA. Therefore, DMUs that are more efficient are expected to perform better in the above weight sets in cross efficiency evaluation than DMUs that are less efficient.

II. Proposed Models

We propose a model minimizing the numbers of efficient DMUs (MINEFF) as the secondary objective for each efficient DMU in DEA. Consider an efficient DMU, DMU_0, MINEFF is formulated as follow:

$$Min \sum_{j\in E} z_j \tag{4}$$

$$s.t. \sum_{i=1}^{m} v_i\, x_{i0} = 1, \tag{5}$$

$$\sum_{r=1}^{s} u_r\, y_{r0} = 1, \tag{6}$$

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + h_j = 0, \qquad j \in E, \tag{7}$$

$$h_j - M z_j \le 0, \qquad j \in E, \tag{8}$$

where $z_j \in \{0,1\}, h_j \ge 0, j \in E,$ E is a set which contains all efficient units; $u_r, v_i \ge 0$, $r = 1, \dots, s$, $i = 1, \dots, m$; M is a large positive number. The efficiency of DMU_0 is preserved by (5) and (6). In (7) and (8), if $z_j = 1$, DMU_j is inefficient; and $z_j = 0$, otherwise. The objective function of MINEFF (4) minimizes the number of efficient DMUs.

In determining a second model to find another weight set, since our goal is to find a weight set which is different substantially from the weights obtained from the previous model, therefore; instead of minimizing the number of efficient DMUs as in MINEFF, we propose maximizing the number of efficient DMUs (MAXEFF) as the secondary objective. Therefore, for each DMU_0, the proposed model, MAXEFF, is listed as follows:

$$Max \sum_{j\in E} z_j \tag{9}$$

$$s.t. \sum_{i=1}^{m} v_i\, x_{i0} = 1, \tag{10}$$

$$\sum_{r=1}^{s} u_r\, y_{r0} = 1, \tag{11}$$

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + h_j = 0, \qquad j \in E, \tag{12}$$

$$h_j - M z_j \le 0, \qquad j \in E, \tag{13}$$

where $z_j \in \{0,1\}, h_j \ge 0, j \in E,$ E is a set which contains all efficient units; $u_r, v_i \ge 0$, $r = 1, \dots, s$, $i = 1, \dots, m$; M is a large positive number. The efficiency of DMU_0 is preserved by (10) and (11). In (12) and (13), if $z_j = 1$, DMU_j is inefficient; and $z_j = 0$, otherwise. The objective function of MAXEFF (9) maximizes the number of efficient DMUs.

In cross-evaluations, it is better to have optimal weight sets that are heterogeneous. Then the efficiency of each DMU can be evaluated by different weight patterns. Increase the heterogeneity among the optimal weight sets used may increase the accuracy of cross-evaluation since for a truly efficient DMU, it is expected that it remains efficient or near efficient under different weight patterns. Our approach provides two sets of optimal weights which are obtained from two opposite secondary objectives. The two sets of optimal weights obtained from the two secondary objectives are expected to be very different in terms of their weight patterns. It should be noted that the proposed approach only uses weight sets obtained from efficient DMUs. We do not use weight sets obtained from inefficient DMUs, since we believe that the weight sets from inefficient DMUs are unlikely to be good representatives of the true underlying weight structure. Therefore, for each efficient DMU, the proposed approach provides two sets of optimal weights for the computations of cross-efficiency ratios.

We summarize our proposed approach as follows:

(1) Determine efficient DMUs in E by solving the CCR model for each DMU.
(2) Solve MINEFF and MAXEFF for all DMUs in E found in Step (1).
(3) Calculate for each DMU, the cross-efficiency ratio using the optimal weight set obtained from MINEFF for each efficient DMU in E. Repeat the computations using optimal weight set obtained from MAXEFF.
(4) Then for each DMU calculate the average cross-efficiency ratio by taking the average of all the cross-efficiency ratios obtained in Sep (3) above.

We call our proposed method combined cross-efficiency approach (COMBINE).

III. Computational Examples

A. *Original Data*

We apply the proposed approach to a data set which was initiated by [14] and had been studied by [15-17]. This data set is composed of one input, which is total cost (TC), and three outputs, namely: the number of teaching units (TU), regular patients (RP) and severe patients (SP) of 15 hypothetical hospitals. The original data were generated by the equation suggested by [14] as shown below:

$$\text{Total Cost} = 0.5\ \text{TU} + 133.68\ \text{RP} + 174.74\ \text{SP} \quad (14)$$

The first seven hospitals were generated as efficient hospitals. The total cost of each efficient hospital is exactly equal to the weighted sum of outputs as computed by (14). The rest were generated as inefficient hospitals. The total cost of each inefficient hospital is larger than the weighted sum of outputs as computed by (14). The inputs, outputs, and true efficiency ratios of the 15 hospitals are listed in Table I. True efficiency ratio of a hospital is derived as the weighted sum of outputs divided by the weighted sum of inputs of the hospital using weights from (14). The first seven hospitals have efficiency ratios equal to one while the efficiency ratios of other hospitals are less than one. We applied DEA (i.e., CCR model) to solve the original 15 hospitals problem. We also applied simple cross-evaluation (SCE) to calculate the average cross-efficiency ratio for each hospital. In SCE, we used weight sets of all DMUs from the CCR model to compute cross-efficiencies of DMUs. Then we solved MINEFF and MAXEFF for each efficient DMU in *E*. We then used optimal weight sets obtained from MINEFF and MAXEFF of each efficient DMU to compute the cross-efficiency ratios for each DMU. Then the COMBINE average cross-efficiency ratio of each DMU was calculated by taking the average of all the cross-efficiency ratios obtained from the two secondary objectives of all the efficient DMUs.

TABLE I. INPUTS AND OUTPUTS OF 15 HOSPITALS

Hospital	TU	RP	SP	TC	True Ratio
1	50	3	2	775.5	1
2	50	2	3	816.6	1
3	100	2	3	841.6	1
4	100	3	2	800.5	1
5	50	3	3	950.3	1
6	100	2	5	1191.05	1
7	50	10	2	1711.3	1
8	100	3	2	884.75	0.9048
9	50	2	3	841.6	0.9703
10	100	10	2	2036.3	0.8527
11	50	5	3	1362.6	0.8936
12	100	3	3	1070	0.9115
13	50	4	5	1491.1	0.9613
14	50	3	2	898.7	0.8629
15	100	3	3	1070	0.9115

TABLE II. RATIOS AND AVERAGE CROSS-EFFICIENCY RATIOS OF DIFFERENT METHODS (15 HOSPITALS, ONE INPUT)

Hospital	True Ratio	DEA Ratio	SCE Ratio	COMBINE Ratio
1	1.00000	1.00000	0.84154	0.99992
2	1.00000	1.00000	0.82983	1.00000
3	1.00000	1.00000	0.92028	0.99967
4	1.00000	1.00000	0.93626	0.99958
5	1.00000	1.00000	0.81951	0.99997
6	1.00000	1.00000	0.86213	0.99996
7	1.00000	1.00000	0.79506	0.99993
8	0.90479	0.90477	0.84711	0.90439
9	0.97027	0.97029	0.80518	0.97029
10	0.85266	0.92333	0.71573	0.85248
11	0.89360	0.89515	0.71999	0.89361
12	0.91145	0.91144	0.81836	0.91121
13	0.96131	0.97252	0.75933	0.96146
14	0.86293	0.86291	0.72617	0.86284
15	0.91145	0.91144	0.81836	0.91121
Pearson Correlation with true ratios		0.946	0.682	1.000
Spearman's rho with true ratios		0.936	0.692	0.949

The results are reported in Table II. In order to measure how well each approach can generate efficiency ratios of each DMU similar to the true efficiency ratios, we computed Pearson correlation and Spearman’s rho between the efficiency ratios obtained from each approach and the true efficiency ratios. We expected that the higher the correlation coefficients, the more similar are the efficiency ratios to the true efficiency ratios.

In Table II, efficiency ratios obtained from DEA and COMBINE have very high Pearson correlations and Spearman’s rho with the true efficiency ratios. However, the ratios obtained from SCE, on the other hand, has low correlations with the true efficiency ratios. This finding can be explained by the effect of alternate optimal solutions discussed previously. The optimal weight set of an efficient DMU obtained from the CCR model is usually coming from the first solution found by the computer software, which very often does not represent the true underlying weight structure. Using weight sets obtained from MINEFF and MAXEFF, average cross-efficiency ratios of COMBINE have high Pearson correlations and Spearman's rho with the true efficiency ratios.

B. Modified Data

The original data set has only one input. However, in DEA, DMUs usually have multiple inputs. As a result, we partitioned the total cost in equation (14), into two elements using the following equation,

$$\text{Total Cost} = 25\ \text{Cost}_1 + 50\ \text{Cost}_2 \quad (15)$$

Substituting (15) into (14), it becomes,

$$25\text{Cost}_1 + 50\text{Cost}_2 = 0.5\text{TU} + 133.68\text{RP} + 174.74\text{SP} \quad (16)$$

TABLE III. INPUTS AND OUTPUTS OF 17 HOSPITALS

HOSPITAL	TU	RP	SP	Cost_1	Cost_2	True Efficiency Ratio
1	50	3	2	11	10.01	1
2	50	2	3	22	5.332	1
3	100	2	3	16	8.832	1
4	100	3	2	15	8.51	1
5	50	3	3	27	5.506	1
6	100	2	5	31	8.321	1
7	50	10	2	28	20.226	1
8	100	3	2	23	6.195	0.9048
9	50	2	3	19	7.332	0.9703
10	100	10	2	30	25.726	0.8527
11	50	5	3	21	16.752	0.8936
12	100	3	3	13	14.9	0.9115
13	50	4	5	24	17.822	0.9613
14	50	3	2	16	9.974	0.8629
15	100	3	3	25	8.9	0.9115
16	100	3	5	20	15.17	1.0526
17	50	4	2	15	14.321	0.8333
Weights	0.5	133.7	174.7	25	50	

The values of the two new costs and the three outputs are listed in Table III. The original total cost of each hospital remains the same after split. The true efficiency ratios of all hospitals are shown in Table III. The first seven hospitals have efficiency ratios equal to one and HOSPITAL_8 to HOSPITAL_{15} have efficiency ratios less than one. The true efficiency ratios of the 15 hospitals are the same as in Table I.

In addition, we added two new hospitals to the data set. Both HOSPITAL_{16} and HOSPITAL_{17} are newly added hospitals. HOSPITAL_{16} is designed to be more efficient than the original 15 hospitals with true efficiency ratio equal to 1.0526, while HOSPITAL_{17} is designed to be less efficient than the original 15 hospitals with efficiency ratios equal to 0.8333. We performed DEA, SCE, and COMBINE on the first 15 DMUs in Table III. The results are reported in Table IV. We first study the original 15 DMUs since their results can be compared with those obtained in Table II and from other previous studies. We then applied all methods to all 17 hospitals. The results are reported in Table V and Table VI.

TABLE IV. RATIOS AND AVERAGE CROSS-EFFICIENCY RATIOS (CER) OF DIFFERENT METHODS (15 HOSPITALS, TWO INPUTS)

Hospital	True Ratio	DEA Ratio	SCE Ratio	COMBINE Ratio
1	1.00000	1.00000	0.83872	0.98393
2	1.00000	1.00000	0.69721	0.92632
3	1.00000	1.00000	0.88098	0.97610
4	1.00000	1.00000	0.88365	0.99374
5	1.00000	1.00000	0.66677	0.92555
6	1.00000	1.00000	0.75036	0.93230
7	1.00000	1.00000	0.70316	0.97378
8	0.90479	1.00000	0.72207	0.87638
9	0.97027	0.97737	0.70749	0.90813
10	0.85266	1.00000	0.67884	0.85075
11	0.89360	0.90178	0.67895	0.86381
12	0.91145	1.00000	0.90322	0.92084
13	0.96131	1.00000	0.71807	0.90965
14	0.86293	0.86292	0.66829	0.83416
15	0.91145	0.92572	0.71987	0.87449
Pearson Correlation with true ratios		0.592	0.334	0.881
Spearman's rho with true ratios		0.539	0.328	0.918

In Table IV, both DEA and SCE perform poorly in terms of Pearson Correlation (and Spearman's rho) which are equal to 0.592 (0.539) and 0.334 (0.328), respectively, while the Pearson Correlation and Spearman's rho of COMBINE are equal to 0.881 and 0.918, respectively. When compared with the results from Table II, the poor performance of DEA and SCE can be explained by an increase in the feasible region of the LP model when the original input was split into two different cost elements. This allows a higher flexibility in choosing optimal weights in the feasible region; therefore, the model has a higher chance to deviate from the true underlying weight set. In practice, multiple inputs is more common than a single input in DEA applications. In this case, the proposed approach performs much better than DEA and the SCE methods.

TABLE V. RATIOS AND AVERAGE CROSS-EFFICIENCY RATIOS OF DIFFERENT METHODS (17 HOSPITALS, TWO INPUTS)

Hospital	True Ratio	DEA Ratio	SCE Ratio	COMBINE Ratio
1	1.00000	1.00000	0.84606	0.95878
2	1.00000	1.00000	0.69589	0.86207
3	1.00000	1.00000	0.83593	0.94974
4	1.00000	1.00000	0.85302	0.99377
5	1.00000	1.00000	0.68024	0.86739
6	1.00000	1.00000	0.72575	0.86820
7	1.00000	1.00000	0.75807	0.95696
8	0.90479	1.00000	0.69934	0.86739
9	0.97027	0.95237	0.70088	0.84856
10	0.85266	1.00000	0.71161	0.85204
11	0.89360	0.88728	0.70040	0.83045
12	0.91145	1.00000	0.87979	0.91100
13	0.96131	0.93639	0.7265	0.84816
14	0.86293	0.84637	0.67365	0.80729
15	0.91145	0.92572	0.69945	0.84737
16	1.05263	1.00000	0.86347	0.96425
17	0.83333	0.89523	0.70304	0.80467
Pearson Correlation with true ratios		0.404	0.468	0.714
Spearman's rho with true ratios		0.639	0.404	0.791

TABLE VI. RANKS OF THE 17 HOSPITALS BY THE DIFFERENT METHODS (TWO INPUTS)

Hospital	True Rank	DEA Rank	SCE Rank	COMBINE Rank
1	5	6	4	3
2	5	6	15	10
3	5	6	5	5
4	5	6	3	1
5	5	6	16	8.5
6	5	6	8	7
7	5	6	6	4
8	13	6	14	8.5
9	9	12	11	12
10	16	6	9	11
11	14	16	12	15
12	11.5	6	1	6
13	10	13	7	13
14	15	17	17	16
15	11.5	14	13	14
16	1	6	2	2
17	17	15	10	17

In Table V, DEA and SCE have Pearson Correlation (Spearman's rho) equal to 0.404 (0.639) and 0.468 (0.404), respectively, while the Pearson Correlation (Spearman's rho) of COMBINE is equal to 0.714 (0.791). Again, COMBINE performs much better than DEA and the SCE methods.

The two added hospitals have true ranks equal to one and seventeen. HOSPITAL_{16} is the most efficient while HOSPITAL_{17} is the least efficient. The results in Table VI indicate that DEA ratios rank HOSPITAL_{16} as 6^{th} and HOSPITAL_{17} as 15^{th}. Since there are eleven hospitals tied in the first place, therefore; HOSPITAL_{16} is ranked as 6^{th} in DEA. The simple cross-evaluation method ranks HOSPITAL_{16} as 2^{nd} and HOSPITAL_{17} as 10^{th}. The proposed approach, COMBINE ranks HOSPITAL_{16} as 2^{nd} and HOSPITAL_{17} as 17^{th}. In the experimental design we intended to make HOSPITAL_{17} as very inefficient compared with other DMUs and it is ranked last among all DMUs in terms of efficiency. However, only COMBINE has ranked HOSPITAL_{17} correctly. Again in this case, the proposed approach performed better than DEA and SCE.

IV. Conclusion

This paper proposed two secondary objectives in DEA to search for better optimal weight sets in the alternate optima in DEA. The two secondary objectives are very different in terms of their optimization objectives: one is to minimize the total number of efficient DMUs and the other is to maximize the total number of efficient DMUs. With these two extremely different objectives, their weight sets obtained are expected to be very different. The intuition of this approach is that a more efficient DMU is expected to perform better than other less efficient DMUs when evaluated with efficient weight sets that are different substantially in their factor weights. Furthermore, adding more weight sets in the computation of cross-evaluation will increase the reliability of the cross-evaluation method. Computational results show that the proposed approach performed well when compared to both DEA and the simple cross-evaluation methods. Future research areas may include the possibility of adding more weight sets that contain factor weights in different weight patterns in the cross-evaluation in DEA.

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