Optimal Incentive Contract to Motivate Task Effort and Mutual Assistance for R&D Project within Virtual Enterprise

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Abstract—This paper studies an incentive contract design problem based on principal-agent theory in the context of virtual enterprise, which considers the mutual assistance behavior of agents and optimizes the total profit of virtual enterprise and completion time of project. Through modeling analysis, it is proved that mutual assistance helps to improve the effort level of agents in virtual enterprise. The results show that mutual assistance among agents in the virtual enterprise is beneficial to revenue increases and efficiency of management work. Under the action of incentive contract, individual's egoistic thought is affected to a certain extent to induce mutual assistance with benefits, thus obtaining greater benefits.

Index Terms—effort levels, R&D, incentive contracts, mutual assistance, virtual enterprise

I. INTRODUCTION

With the rapid development of economic globalization and information technology, the uncertainty and technical risk in the process of product R&D have greatly increased. Under this background, it is difficult for a single enterprise to grasp the ever-changing market opportunities and increasingly fierce market competition by relying on their own ability and resources [1, 2], so as to survive the intensive competition. The rapid development of Internet communication technology makes it possible for enterprises to share information and help each other across regions. Owing to the differences of core competitiveness each enterprise obtained, such as resources, skills, key technologies, and strategies, more and more enterprises choose to establish virtual enterprises to achieve mutual goals to reduce technical risks, utilize superior R&D resources, shorten product R&D cycle, and improve product quality and technical content.

A virtual enterprise usually consists of a core enterprise and several partner enterprises [3-5]. Specially, the core enterprise divides the whole product R&D process into several tasks, and assigns them according to the competitiveness of partner enterprises. However, due to the asymmetric information between the core enterprise and other profit-driven partner enterprises, the core enterprise is at an information disadvantage and the efforts and knowledge factor that partner enterprises devoted in the task are unobservable and unmeasurable [6, 7]. Such asymmetric information can induce the opportunistic behavior of enterprises, which may further affect the profit of the entire alliance [8, 9].

Mutual assistance is conducive to improve the efficiency of virtual enterprise. The impeccable multi-task cooperation and mutual assistance incentive mechanism, which directly affect the collaboration effects and product quality, are an important basis for enterprise alliance to get to the substantive stage [10,11]. Therefore, the virtual enterprise leader needs to develop a reasonable and effective incentive mechanism for mutual assistance, especially under the consideration of the project completion time, resource constraints and the ability level of the alliance member. In this case, mutual assistance behavior can be realized through the specialized division of labor and incentive to reduce opportunistic speculation due to information asymmetry, thus improving the efficiency of company management.

The existing studies on incentive mechanism design mostly focus on the performance incentives for internal employees [11-13], but incentive mechanism for mutual assistance is rarely considered. For example, Feng W [11] studied the employee incentive mechanism of non-public enterprises and established an index evaluation system from the aspects of performance incentive, ability incentive and environmental incentive. Based on the two-factor theory, Wang P, Lu Z N, Sun J H [12] indicated that external incentives positively affected task performance and innovation performance, and then empirically investigated the impact of different incentives on management performance. In addition, Akram E T [14] studied contracts that introduce incentives in the form of bonuses and penalties on the basis of fixed payment contracts and examines their impact on project completion time. Kai W, Lin Y X, Johannes K [15] proposed a new incentive model, which was establish hed on the basis of considering enterprise carbon emission intensity and cooperative game theory.

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As is illustrated above, existing research has limitations on mutual assistance and task effort. We have to approve that solving problems on multi-task cooperation and mutual assistance incentive mechanism indeed has practical significance.

Considering the management meaning and value, the incentive mechanism of multi-task cooperation is studied. This paper proposes an incentive contract design based on the task assignment model of mutual assistance which counts the effort level of alliance members. Based on the principal-agent theory, this paper demonstrates the optimal effort level and the relationship between task load and the optimal benefit by modeling. Moreover, in order to encourage all partners to make efforts and ensure the total profit of enterprise alliance, we solve the optimal incentive contract based on cooperation in the process of product development.

The reminder of this paper is organized as follows: Section 2 proposes an incentive contract model and derives the benefit functions of both principal and agents. Section 3 discusses the process of solving the model and related theorems. Finally, Section 4 summarizes the paper and describes the managerial significance.

II. PROCEDURE FOR PAPER SUBMISSION

Consider a virtual enterprise that comprise a core enterprise and two partner enterprises to complete a R&D project. According to the principal-agent theory, we take the core as the principal and the partners as the agents who have different core competitiveness. Considering that the final benefit is related to the completion time, and the profit function is negatively related to the completion time. Thus, it is assumed that the total project revenue function is \( P = \frac{\omega}{T} \), in which \( \omega \) indicates the remuneration the client provides, and \( T \) represents the time required to complete the task given by the client.

According to the type of the task, we divide the task into \( \pi_1 \) and \( \pi_2 \). Assign tasks to agents with corresponding core competence, and \( \pi \) represents the task load. Assuming that capacity level of the two agents toward corresponding task as \( \mu_1 \) and \( \mu_2 \) respectively. Mutual assistance behavior which speeds up the progress of the task and improves the efficiency between agents. Without loss of generality, agent 2 provides assistance to agent 1 during the conduction of the R&D project. Here we assume \( \mu_b \) for the competence level of agent 2 offers to the other.

Actually, the output of an agent is related to its effort level, so the task completion time can be given as

\[
t_1 = \frac{\pi_1}{\mu_1 e_1 + \mu_b e_2}
\]  
(1)

The principal provides agent 1 with output sharing incentive mechanism \( S_1 = a_0 + a_1 P \), where \( a_0 \) is the fixed income of agent 1 promised by the principal, and \( a_1 \) is the shared output. The effort cost of agent 1 is monetized as \( C_1 = \frac{e_1^2}{2\mu_1} \). Similarly, provide agent 2 with output sharing incentive mechanism \( S_2 = b_0 + b_1 P \), where \( b_0 \) is the fixed income of agent 2 promised by the principal, and \( b_1 \) is the share of output. The effort cost of agent 2 is monetized as \( C_2 = \frac{e_2^2}{2\mu_2} + \frac{e_1^2}{2\mu_b} \).

In the optimization model, to highlight the role of mutual assistance and effort levels at the R&D project, the influence of random factors on the income is ignored, only the income of the agent is considered:

\[
U_i = S_i - C_i \quad (i = 1, 2)
\]  
(3)

According to the previous research and calculation, if \( t_1 < t_2 \), it will get conclusions that have no reference value. On the other hand, from the practical consideration, agent 2 has the behavior of helping agent 1, so agent 2 has to complete the assigned task in advance, namely, \( T = t_1 \). The expected revenue functions of agent 1 and agent 2 can be expressed as follows:

\[
U_1 = a_0 + \frac{a_1 \omega (\mu_1 e_1 + \mu_b e_2)}{\pi_1} - \frac{e_1^2}{2\mu_1}
\]  
(4)

\[
U_2 = b_0 + \frac{b_1 \omega (\mu_1 e_1 + \mu_b e_2)}{\pi_1} - \frac{e_2^2}{2\mu_2} - \frac{e_1^2}{2\mu_b}
\]  
(5)

The higher competence level of agent to corresponding task, the higher the income, and its income is affected by their respective effort level, providing that effort level can improve the final income. The corresponding expected revenue of the principal can be expressed as:

\[
\max_{\mu_i, \mu_j} U_p = P - \sum_{i=1}^{2} U_i \quad (j = 0, 1)
\]  
(6)

As such, given the effort of the agents, the principal can choose the appropriate incentive contract coefficient to improve his own expected revenues.

III. MODEL SOLUTION AND ANALYSIS

According to the incentive contract based on the effort level and mutual assistance behavior, in the R&D project, the agent will first consider what level of effort can he obtain the maximum benefit, and agent 2 will also
consider to help agent 1 with what level of effort.

**Proposition 1** From the view of agents, when their effort levels are $e_1^*, e_2$ and $e_h^*$, they have the optimal expected revenue $U_1^*$ and $U_2^*$.

1. $e_1^* = \frac{a_1 \omega \mu_1^2}{\pi_1}$, $e_2^* = \frac{\pi_2}{\pi_1 \mu_2}$

2. $U_1^* = a_0 + \frac{a_1 \omega^2 (a_1 \mu_1^3 + 2 b_1 \mu_1^3)}{2 \pi_1^2}$


$U_2^* = b_0 + \frac{b_0 \omega^2 (2 a_0 \mu_1^3 + b_1 \mu_1^3)}{2 \pi_1^2} - \frac{\pi_2^2 \omega^2 (a_1 \mu_1^3 + b_1 \mu_1^3)^2}{\pi_1^2 \mu_2^2}$

**Proof:** By analyzing the expected revenue formula (4) of agent 1, it can be obtained that agent 1 can control its expected revenue is optimal, $\frac{\partial U_1}{\partial e_1}$, namely, $e_1^* = \frac{a_1 \omega \mu_1^2}{\pi_1}$. Therefore, the optimal expected revenue of agent 1 is

$$U_1^* = a_0 + \frac{a_1 \omega^2 (a_1 \mu_1^3 + 2 b_1 \mu_1^3)}{2 \pi_1^2}$$

Analyzing the expected revenue formula (6) of agent 2, it can be concluded that the revenue of agent 2 is affected by its effort level and the effort level on assisting. When the expected revenue is optimal, $\frac{\partial U_2}{\partial e_h}$, that is, $e_h^* = \frac{b_0 \omega \mu_2^2}{\pi_1}$.

It is known above that $l_1 > l_2$, namely, $e_2 \geq \frac{\pi_2}{\pi_1 \mu_2} (\mu_1 e_1 + \mu_2 e_2)$. From formula (5), the expected revenue of agent 2 decreases with the increase of $e_2$. Therefore, $e_2^* = \frac{\pi_2}{\pi_1 \mu_2} (\mu_1 e_1 + \mu_2 e_2)$. Then the optimal expected revenue of agent 2 is

$$U_2^* = \frac{b_0 \omega^2 (2 a_0 \mu_1^3 + b_1 \mu_1^3)}{2 \pi_1^2} - \frac{\pi_2^2 \omega^2 (a_1 \mu_1^3 + b_1 \mu_1^3)^2}{\pi_1^2 \mu_2^2} + b_0$$

Incentive contracts based on effort levels make agents willing to put effort into R&D project, and agents can independently adjust their own efforts to affect their expected revenues, which is not completely determined by the principal. Such incentive contract setting enables agents to participate in the decision-making of their own incomes, which is conducive to improving their enthusiasm for production and research, and improving the completion efficiency and the quality of tasks.

The incentive contract based on effort level, actually, can optimize the expected revenue of the principal when motivate the agent to put efforts into R&D project. Especially, the principal can predict the effort level of agents, and then develop appropriate incentive contract coefficients to improve his own revenue.

**Proposition 2** Given the effort level $e_1^*, e_2^* , e_h^*$ of agents, the principal can choose $a_1^*$ and $b_1^*$ as the coefficients of incentive contract, so as to obtain the optimal expected revenue $U_p^*$.

1. $a_1^* = 1 \left(1 + \frac{\mu_1^3}{\mu_0} + \frac{\mu_1^3}{\mu_0} - 2 \pi_1^2 \frac{\omega^2}{\mu_1^2} (\mu_1^3 + \mu_0^3) \right)$

2. $b_1^* = 1 \left(1 + \frac{\mu_1^3}{\mu_0} + \frac{\mu_1^3}{\mu_0} - 2 \pi_1^2 \frac{\omega^2}{\mu_1^2} (\mu_1^3 + \mu_0^3) \right)$

**Proof:** By sorting out formula (7), (9) and (13), the expected revenue of the principal can be expressed as

$$U_p^* = \frac{\omega^2 (a_0 \mu_1^3 + b_1 \mu_1^3)}{\pi_1^2} (1- a_0 - b_0) + \frac{\omega^2 (a_1 \mu_1^3 + b_1 \mu_1^3)}{2 \pi_1^2}$$

$$+ \frac{\pi_2^2 \omega^2 (a_1 \mu_1^3 + b_1 \mu_1^3)^2}{\pi_1^2 \mu_2^2} - a_0 - b_0$$

Therefore, the income of the principal is affected by the incentive contract parameters $a_0, a_1, b_0$ and $b_1$. According to formula (18), $U_p^*$ has a linear relationship with $a_0$ and $b_0$, and decreases with the increase of $a_0$ and $b_0$. Therefore, when the principal achieves the optimal expected revenue, $a_0 = b_0 = 0$, and $\frac{\partial U_p}{\partial a_1} = \frac{\partial U_p}{\partial b_1} = 0$, the combined solution is obtained.

$$a_1^* = \frac{1}{1 + \frac{\mu_1^3}{\mu_0} + \frac{\mu_1^3}{\mu_0} - 2 \pi_1^2 \frac{\omega^2}{\mu_1^2} (\mu_1^3 + \mu_0^3)}$$

$$b_1^* = \frac{1}{1 + \frac{\mu_1^3}{\mu_0} + \frac{\mu_1^3}{\mu_0} - 2 \pi_1^2 \frac{\omega^2}{\mu_1^2} (\mu_1^3 + \mu_0^3)}$$

Substituting formula (10) and (11) into formula (9), the optimal solution of the principal can be obtained. The design of incentive contract in this paper can
enable the principal to make further decisions based on predicting the effort level of agents, and optimize its own revenue by adjusting the contract coefficient. It helps the principal to occupy an advantageous position in information acquisition and establish the core position of decision-making.

Recalling the proposition 1, we found a relationship between the optimal effort level and the task load. Obviously, the task load can also affect the expected revenue of the agents and principal.

**Proposition 3** The expected revenue \( U_p \) decreases with the increase of task load \( \pi_1 \) and \( \pi_2 \).

**Proof:** As the revenue function of the principal is too complicated, a benchmark is set here for further study, and it is better to assume that the competence level of the agents reaches the maximum value (\( \mu_1 = \mu_2 = \mu_n = 1 \)), at this rate,

\[
U_p = \omega^2 \left( \frac{1}{\pi_1^2} - \frac{3}{\pi_2^2} - 4\pi_i^2 \right) \quad (12)
\]

Take \( U_p \) partial respect to \( \pi_1 \) and \( \pi_2 \), you can get a function which always less than 0. It is not difficult to find that the actual profit of the principal decreases with the increase of \( \pi_1 \) and \( \pi_2 \), and the influence degree is different.

Combined with the proposition 1, the task load of agent 1 is negatively correlated with the effort level of two agents, and the increased task load of agent 2 can actually motivate him to make more efforts in mutual assistance. However, \( \pi_1 \) makes a bigger impact at effort level. Under the premise of a certain total amount of tasks, the influence degree of \( \pi_1 \) and \( \pi_2 \) on the revenue of the principal is different, so the principal can optimize the expected revenue by adjusting the amount of tasks assigned to each agent, which further reflects the core position of the principal in decision-making. In R&D projects, the impact of task load on revenue is often ignored by managers. The study on task load in this paper should be of great reference significance for today's incentive management.

This paper mainly considers the R&D project under the mutual assistance. In order to enhance the accuracy and persuasion of the research, we also need to analyze the revenues of principal and agents in R&D project which mutual assistance does not exist. And the exact conclusion is obtained by comparison.

**Proposition 4** As mutual assistance exists among agents, the expected revenues of principal and agents will all increase.

**Proof:** Without considering the mutual assistance of agents (i.e., when \( e_n = 0 \)), since the size relationship of \( t_1 \) and \( t_2 \) does not hinder the calculation of this part, \( t_1 > t_2 \) could be set, as well

\[
U_1^* = a_0 + \frac{a_1^2 \omega^2}{2 \pi_1^2} \quad (13)
\]

\[
U_2^* = b_0 + \frac{b_1^2 a_1^2 \mu_1}{\pi_1^2} - \frac{\pi_1^2 \omega^2 (a_1 \mu_1^3)^2}{\mu_2^2} \quad (14)
\]

Substitute formula (13) and formula (14) into formula (6) to obtain the expression of optimal revenue of the principal:

\[
U_p = \frac{a_1 \mu_1 (1-a_2-b_1)}{\pi_1^2} + \frac{\omega^2 a_1 \mu_1}{2 \pi_1^2} + \frac{\pi_1^2 \omega^2 (a_1 \mu_1^3)^2}{\mu_2^2} - a_0 - b_0 \quad (15)
\]

By comparing formula (7) and (13), (8) and (14), and (9) and (15), it is not difficult to see that when there is mutual assistance among agents, the revenues of agents and principal will increase.

Based on effort level and mutual assistance, the incentive contract enables an agent to provide assistance to another agent after he or she has made efforts to complete the task, so as to shorten the completion time and improve the revenue. The contract established in this paper induces agents to conduct mutual assistance through interests, so as to improve the revenue of the principal, thus forming a virtuous circle. However, today’s R&D projects focus on the ability level of agents, often ignoring the fact that mutual assistance between agents to improve performance, resulting in agents ignoring the overall progress of the task, greatly extending the completion time. Therefore, the incentive contract based on mutual assistance in this paper plays a great role in improving the current R&D projects and management mode. Managers should learn from this model and guide agents to form mutual assistance.

**IV. CONCLUSION**

Based on the task assignment model of mutual assistance, this paper proposes an incentive contract design that considers the effort level of agents. Based on the principal-agent theory, the relationship between the optimal effort level and the task load and the optimal expected revenue of the principal and the agents is demonstrated.

The main contributions of this paper are as follows:

1. From the perspective of agents, when their effort level are \( e_1^* \), \( e_2^* \) and \( e_n^* \), agents have optimal expected revenues. This means that the agents can adjust their own efforts to affect their revenues, not completely decided by the principal. (2) The principal can choose \( a_1^* \) and \( b_1^* \) as the coefficient of incentive contract, so as to
obtain the optimal expected revenue $U_p$. Therefore, the principal is in a dominant position when making decision and can stabilize the enterprise alliance. (3) The expected revenue of the principal decreases with the increase of $\pi_1$ and $\pi_2$. In the case that the total amount of tasks is fixed, task assignment can be adjusted to achieve the purpose of improving revenue. (4) Mutual assistance exists among agents, the revenue of principal and agents will all increase. The incentive contract based on mutual assistance plays a great role in improving the current R&D projects and management mode.

The contribution of this paper should be evaluated along with certain limitations. This paper does not consider the problem of opportunism caused by asymmetric information in subsistent enterprise alliance. Considering the problem of incentive contract setting in the case of asymmetric information deserves to be studied and it will be our future research direction.

CONFLICT OF INTEREST
The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS
Li conducted the research and built up the model; Geng collected the literatures and analyzed the background; Sha assisted to write and modify the paper; all authors had approved the final version.

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REFERENCES