Stability and Its Determinants of Platform Ecosystem

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Abstract—**How can stability help platform to thrive? In management sciences, traditional studies have based on theoretical concepts of platforms ecosystem and its stability conditions, but no one have found numerical answers. Suppliers-Consumers mutualistic network sustains the needs of suppliers and consumers in daily life. Global economic changes threaten these networks to move from stable state to the instable one and creating perturbations inside platforms. Here we developed a mathematical model (S-C model) which incorporating the dynamics, interactions and mutualistic network for platform ecosystem. This model served to us to predict the system's stability conditions, our predictions can be calculated using the reduced model (with artificial data). But we faced with difficulty of random variables that made the system more complex, so to better achieve our results we used the Monte Carlo method to simulate the variables and find out their confidence interval that contribute to the stability conditions. Our model can serve as a paradigm to understand and control the stability of platforms in real mutualistic networks for the safeguard of platforms. The general principle can be extended to a wide range of disciplines to deal with stability issues**.

Index Terms—**Stability, Platform ecosystem, Consumers and suppliers, Interactions, Collapse, Monte Carlo**

I. Introduction

A platform based ecosystem is as a network where platform owners promotes others to develop complementary innovations, the resulting corporate network shows great *i* interdependencies [1, 2, 3]. It is a two-sided market, an established environment to allow multiple groups of users (suppliers and consumers) to exchange views for fair deals [4]. Two sided-markets involve two groups of agents (suppliers and consumers) where the benefits gained by one group from joining the platform depends on the size of the other group joining that Platform [5]. We find platforms in many industries such as Google, Facebook, video games, payment cards,…, etc. These examples are all industry platforms [6]. Recently, companies from various industries have identified new competitive advantages by creating compelling experiences for their customers. The intense competition within companies is increasingly concentrated on platforms [6]. The world has seen the emergence of many digital platforms, challenging companies, and they have established themselves as market leaders in just a few years [7]. Platforms compete with each other to ensure stability, which makes them more innovative and more likely to convince users to adopt them [8]. Stability in the platform ecosystem is important to counteract the disruptions that occur.

Many complex dynamical systems such as in ecology, earth and environmental science, economy, astronomy,..., etc have attached a great importance to the stability of their systems [9, 10, 11, 12, 13, 14, 15]. Some examples of using stability to reduce the risk of systems collapses: The concept of stability for reducing risks of collapsing is used in environmental science to avoid the risk of soil collapse [12]. Zahid Hussain stated that economic growth and political stability are closely linked. The uncertainty associated with an unstable political environment can lead to sluggish investment and growth. Therefore, weak economic performance may lead to the collapse of the government and political turmoil. We see also in [14] that the overturn stability is a critical safety concern for platforms like swinging cranes. In economy and management science when the outputs of a system are under control it is said to be stable. Here we base on platforms ecosystem and its stability because they have had a huge impact on business over the past two decades [16].

Stability is the lifeblood for the smooth functioning of any platform. The platforms stability measures the balancing capacity, which is essential to properly manage the platforms activities and cope with certain disturbances that can occur at any time [10]. The instability limits the system to predict the future perturbations and eventually drive towards platform collapse. Therefore it is important to find out conditions that can cause uncertainty and fails

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the platform. When the outputs of a system are under control, it is said to be stable. It identifies the risks and avoids unexpected structural collapse during changes. Platform overturn stability is an important safety factor [11]. Empirical studies indicated those platforms ecosystems are more instable than what are the theoretical studies show [17]. Many papers talked about the importance of stability of the platforms but no one modelize it and analyses it numerically in the same way as we did. In our study we analyse numerically the stability of platforms ecosystem.

To define the generic model of the platform ecosystem, we take help from a Lotka-Voltera model based on [18, 19, 20] (Models based on differential equations attempt to understand the relationships of prey and predators and to characterize the mechanics of biological systems [21], but they can also be generalized to the case of a group of suppliers and consumers). Suppliers need to know how consumers will react toward their products so they can sell them effectively. The study of consumer behavior consists of examining which products they prefer to buy and when and how consumers react to these products. Understanding consumer behaviors towards these products will help to grow the business by meeting their needs [22]. Consumers naturally tend to change their suppliers and products; this makes them feel more satisfied and confidence [22]. Consumers place a great value on those platforms with more users (because of direct network effect), they think that those platforms will offer to them a large and more divers of services (what we call indirect network effect) [23]. This makes the interactions in a platform more complex. Researchers in various disciplines such as psychology, mathematics, and management focus on the relationship between suppliers and consumers.

We build up a large number of connected users (set of interacting elements). If the interacting components are globally dynamic, then it makes the system more complex [24, 25]. Our model represents both suppliers and consumers growth (S-C model), mainly focusing on the mutual interaction within suppliers (the alliance partnership). A commercial alliance is an agreement between companies to exchanges services. For example, Uber and music streaming service Spotify has announced a partnership that will enable Uber passengers to listen to their own Spotify playlists during car journeys. This study also considered competition within suppliers. Because competition is a factor that greatly affects the growth of platforms either negatively or positively, it can be either helpful or be fatal for platforms growth. The increasing competition in the platform motivates the companies to improve creativity in production (innovation); otherwise, they gradually lose their market share, leading to a shutdown. For example, Samsung, Apple, and Nokia have their own system, and their competition is very sharp. The continuous innovation in Samsung and Apple products increases their share in the platform while Nokia almost disappeared. In our S-C model, we set the negative interaction within consumer (consumers are attracted to the best offers from suppliers, so they always try to take advantage of the offer, if they don't, most of the time, they share tier bad experience which will affect negatively the growth of the number of consumers in the platform). Finally, we consider the mutualistic interaction between suppliers and consumers more important.

The competition within the platform can be likened to predators and prey; the difference is that the competing companies are predators and preys simultaneously. Each one tries to destroy the other. While the alliance can be compared to plants and pollinators, the plant needs pollinators either pollinators do not need the plant. However, in the alliance of companies representing mutual benefit, each one needs the others.

We use the Lyapunov theory of stability (the most popular method in determining control system's stability [26]), to start with, we analyze in which areas the Eigen values are negative (stable state areas). To enhance our stability results we take into consideration the diversity inside platform. A growing number of empirical studies demonstrate positive diversity-stability relationships [15]. Greater diversity leads to greater stability. The overall diversity of an ecosystem is often the determinant of stability against different perturbations [15]. We rely on ecosystems and inspired ideas from them. The variables involved in this system are random, which makes it difficult to predict. Therefore, this study uses the Monte Carlo method to simulate them and determine their confidence interval (safe interval). We tested the interaction between suppliers and consumers in the network and found out its probability. We concluded that more interactions lead the system towards stabilization. More interaction is directly related to the number of consumers visiting the platform; whenever there are many visitors, the consumer's probability of interacting with suppliers becomes very high. This study built a dynamic system model based on the researchers of [18, 19] to understand the mechanism and growth patterns of the platform ecosystem. A system is dynamic if it includes a phase space E whose elements represent the states of the system, time t that can be discrete or continuous, and the law of evolution. In general, knowing the state at time t_0 makes it possible to determine the state at any time $t > t_0$ [24]. Platforms ecosystems are difficult to control because they are in the form of complex systems. This research is intended to predict the systems stability conditions that will help in the platforms evolution and aid in avoiding platform collapse.

II. Methodology

A. Model Structure

The model developed in this research is a pair of non-linear differential equations of the first order. It describes the dynamics of our system, including different interactions. The model developed in this study is based on the Lotka-Volterra approach, which also a couple of differential equations of first order but used to describe the dynamics of a biologic system (predator-prey). The

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evolution of the platform ecosystem used in this study is as follow:

$$\begin{cases} \dfrac{dS_i}{dt} = F_1(S_i) + F_2(S_i,\ C_i) \\ \dfrac{dC_i}{dt} = G_1(S_i) + G_2(S_i,\ C_i) \end{cases} \quad (1)$$

The first terms of (1) ($F_1(S_i)$ and $G_1(C_i)$) describes the self-dynamic of each component (suppliers and consumers, respectively). The second term ($F_2(S_i,C_i)$ and $G_2(S_i;C_i)$) describes the interactions between suppliers and consumers; where S_i and C_i are the number of suppliers and consumers, respectively, in the group *i*. Our model is based on the form of Lotka-Volterra, where:

$$\begin{cases} F_1(S_i) = S_i\left(r_i - \sum_{j\in M_1} B_{ij}S_j + \sum_{j\in M1} \delta_{ij}S_j\right) \\ G_1(C_i) = C_i\left(\mu_i - \sum_{j\in M_1} \lambda_{ij}C_j\right) \end{cases} \quad (2)$$

And

$$\begin{cases} F_2(S_i,\ C_i) = S_i \dfrac{\sum_{k=1}^{M_2} \gamma_{ik}^{(S_i)} C_k}{1 + h\sum_{k=1}^{M_2} \gamma_{ik}^{(S_i)} C_k} \\ G_2(S_i,\ C_i) = C_i \dfrac{\sum_{k=1}^{M_1} \gamma_{ik}^{(C_i)} S_k}{1 + h\sum_{k=1}^{M_1} \gamma_{ik}^{(C_i)} S_k} \end{cases} \quad (3)$$

Here we have S_i and C_i are the number of suppliers and consumers (respectively), r_i represent the growth rate for each supplier in group *i* (without intraspecific and interspecific competition), this rate to the percentage change of the number of suppliers within time. μ_i the growth rates of consumers (without intraspecific and interspecific competition), B_{ij} represent the coefficient of competition between supplier in group *i* and suppliers in group *j*; δ_{ij} is the coefficient of mutualistic interaction (or we can say the alliance or partnership) between suppliers in group *i* and suppliers in group *j*; λ_{ij} is Coefficient of the negative interaction within consumers (for example when a consumer *i* share his bad experience in the platform many other consumers' will drop out from the platform (number of consumers will decrease) which will hinder the growth of the platform); $\gamma_{ik}^{(S_i)}$ and $\gamma_{ik}^{(C_i)}$ the strength of mutualistic interaction coefficient between consumers and suppliers (in the side of suppliers and consumers respectively), *h* is the Half-saturation point (is a constant which limits the number of suppliers and consumers [8]) because of mutualistic interactions between suppliers and consumers is almost growing up to the infinity, it must be a sauration. The generic model is in the form:

$$\begin{cases} \dfrac{dS_i}{dt} = S_i\left(r_i - \sum_{j\in M_1} B_{ij}S_j + \sum_{j\in M_1} \delta_{ij}S_j + \dfrac{\sum_{k=1}^{M_2} \gamma_{ik}^{(S_i)} C_k}{1 + h\sum_{k=1}^{M_2} \gamma_{ik}^{(S_i)} C_k}\right) \\ \dfrac{dC_i}{dt} = C_i\left(\mu_i - \sum_{j\in M_1} \lambda_{ij}S_j + \dfrac{\sum_{k=1}^{M_1} \gamma_{ik}^{(C_i)} S_k}{1 + h\sum_{k=1}^{M_1} \gamma_{ik}^{(C_i)} S_k}\right) \end{cases} \quad (4)$$

We combine (1), (2) and (3) to get the generic model (4). The competition within suppliers of the same commodities (intraspecific competition) and in same platform is assumed to be stronger than the competition within suppliers of different commodities (interspecific competition) and different platforms [8, 9, 18]. Mathematically we can write: $B_{ii} >> B_{ij}$ [18] and partnership within suppliers in the same commodities is stronger than the partnership within suppliers in different commodities $\delta_{ii} >> \delta_{ij}$ with $i \neq j$, in other side we also assume that the negative interaction in same commodities is stronger than the negative interaction in different commodities within consumers $\lambda_{ii} >> \lambda_{ij}$ with all $i \neq j$. Competition coefficient depends on the competition strength b_{ij} , carrying capacities K_i and growth rate r_i [28], we have B_{ij} in the form: $B_{ij} = b_{ij}.\dfrac{r_i}{K_i}$ we set here b_{ij} = 1 if there is competition and b_{ij} = 0 otherwise. We assume that B_{ii} = 1 and $B_{ij} = \rho_1$ when $i \neq j$. Because also we dont have the empirical informations about the alliance (partnership) between suppliers and the negative interaction within consumers we assume that: $\lambda_{ii} = \rho_2$ and $\lambda_{ij} = a_1$ if $i\neq j$ and $\delta_{ii} = \rho_3$, $\delta_{ij} = a_2$ if i≠j. We suppose that $\rho_1 \in [0,1]$, $\rho_2 \in [0,1]$, $\rho_3 \in [0,1]$ and a_1, a_2 are constants.

When there is cross-reactivity within partners, it tends to increase the abundance and then when the mutual partners have great abundance, the beneficial effect of interactions on the growth of partners will be saturated. When the strength of the mutualistic interaction equal to zero ($\gamma_{ij} = 0$) means that no interaction in the network. But in general it depends on the degree of node and: $\gamma_{ij}^{(S_i)} = \varepsilon_{ij}\dfrac{\gamma_0}{(G_i)^l}$ and $\gamma_{ij}^{(C_i)} = \varepsilon_{ij}\dfrac{\gamma_0}{(Z_i)^l}$. Here γ_0 is

The average mutualistic strength, we assume that $\gamma_0 = 1$. We have if : ε_{ij}=1 means there is interaction in the network and if ε_{ij}=1 otherwise; G_i and Z_i are the numbers of interactions that benefit from the interactions in the two sides respectively (suppliers and consumers); *l* represents the length trade-off between the interaction strength γ_{ij} and the number of interactions G_i and Z_i (A trade-off is a situational decision that involves diminishing or losing one quality, quantity or property of a set or design in return for gains in other aspects) between the interaction strength and the number of interactions. We distinguish two neural cases:

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- If l = 0: means that the network topology will have no effect on the strength of the mutualistic interactions (no tradeoff).
- If l = 1: means that the network topology will affect the suppliers gain from the interactions (full tradeoff).

The starting point of our mathematical analysis of our system's stability is to get the reduced mode. In the supplementary information we detail the steps of our dimension reduction procedure (based on [8]), which leads to the reduced model:

$$\begin{cases} \dfrac{dS_{eff}}{dt} = rS_{eff} - BS_{eff}^2 + \delta S_{eff}^2 + \dfrac{\left\langle \gamma^{(S_i)} \right\rangle C_{eff}}{1+h\left\langle \gamma^{(S_i)} \right\rangle C_{eff}} S_{eff} \\ \dfrac{dC_{eff}}{dt} = \mu C_{eff} - \lambda C_{eff}^2 + \dfrac{\left\langle \gamma^{(C_i)} \right\rangle S_{eff}}{1+h\left\langle \gamma^{(C_i)} \right\rangle S_{eff}} C_{eff} \end{cases} \quad (5)$$

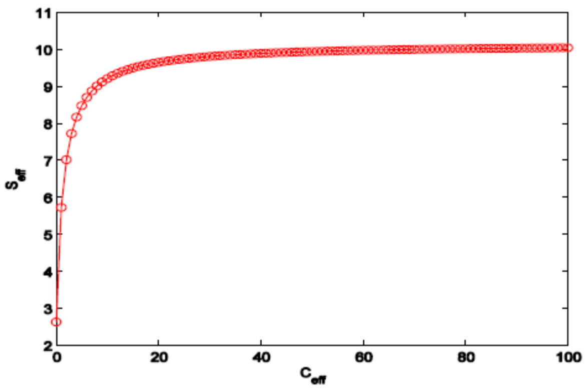
Where the dynamical variables S_{eff} and C_{eff} are the effective or the average number of suppliers and consumers, respectively; r and μ are the effective growth rates for suppliers and consumers, respectively; B is the parameter which characterized the effect of the intraspecific and the interspecific competition; δ is the parameter which characterized the effect of intraspecific and interspecific alliance and λ is the mean number of the negative interaction coefficient within consumers, $\left\langle \gamma_{ij}^{(S_i)} \right\rangle$ and $\left\langle \gamma_{ij}^{(C_i)} \right\rangle$ are the effective mutualistic strength associated to the suppliers and consumers.

The equilibrium or the steady state solution can be obtained by solving:

$$\begin{cases} \dfrac{dS_{eff}}{dt} = 0 \\ \dfrac{dC_{eff}}{dt} = 0 \end{cases} \quad (6)$$

After solving equation.6 we found:

$$\begin{cases} S_{eff} = \dfrac{-\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff} - r\left(1+h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}\right)}{\left(1+h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}\right)(\delta - B)} \\ C_{eff} = \dfrac{-\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff} - \mu\left(1+h\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}\right)}{\left(1+h\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}\right)(-\lambda)} \end{cases} \quad (7)$$



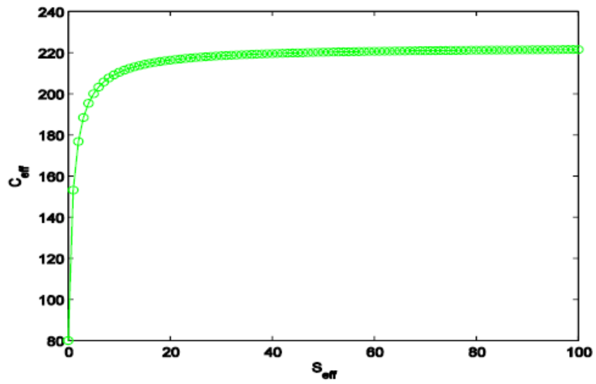


Figure 1. The first graph on the left hand shows the steady state values depending on the variation of the S and C where we set C_{eff} ϵ [0; 100], the second graph on the right hand shows the variables that consumers take depends on the number of suppliers and we set S_{eff} ϵ [0; 100] in both graphs we fixe B_{eff} = 0:2; $\left\langle \gamma^C \right\rangle = 1.5$; $\left\langle \gamma^S \right\rangle = 1$; $r = 0.3$; $h = 0.7$; $\lambda = 0.01$; $\mu = 0.8$; $\delta = 0.01$.

B. *The Stability Conditions*

In order to derive the stability conditions, we start by studying the approximation of linear Lotka-Voltera (h= 0) of our dynamic system (4). For (h= 0) our system can be writeen as follow:

$$\begin{bmatrix} \dfrac{dS}{dt} \\ \dfrac{dC}{dt} \end{bmatrix} = Diag\left(\begin{bmatrix} S \\ C \end{bmatrix}\right) \times \left(\begin{bmatrix} r \\ \mu \end{bmatrix} - \begin{bmatrix} B-\delta & -\gamma^S \\ -\gamma^C & \lambda \end{bmatrix}\right) \quad (8)$$

The interaction strength matrix A is in the form:

$$A = \begin{bmatrix} B-\delta & -\gamma^S \\ -\gamma^C & \lambda \end{bmatrix} \quad (9)$$

a. Special Cases

1- In the case l = 0 and λ is a diagonal matrix, and then the interaction strength matrix A is a symmetrical matrix.

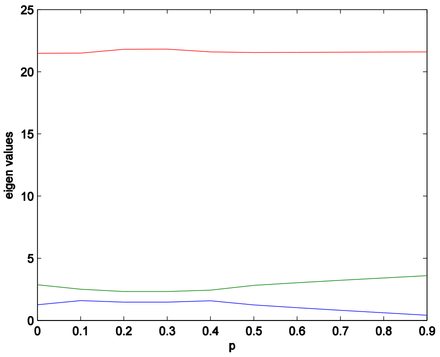
2- In the case $\rho_1 = 0$, the matrix A is a Z-matrix ((matrices whose off-diagonal elements are no positive)

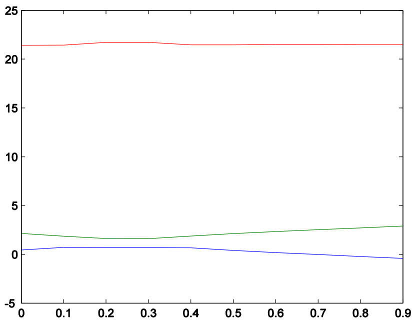
In these two particular cases, it was shown in [29] that the Lyapunov stability and the diagonal Lyapunov stability are equivalent conditions. i.e if the real parts of all the eigenvalues of A are positive then, any achievable equilibrium point is globally stable, (see [30]). Mathematically, it is difficult to prove that a given matrix is D-stable; because there is no mathematical method available to find a positive diagonal matrix D such that $DA + A^t D > 0$. On the other hand, we know for $\rho_1 > 0$ and l > 0, there are no analytical results yet demonstrating that the diagonal stability of Lyapunov is equivalent to the stability of Lyapunov. However, we assume that the two main consequences of Lyapunov diagonal stability are maintained [31]. More precisely, we assume the following conjectures:

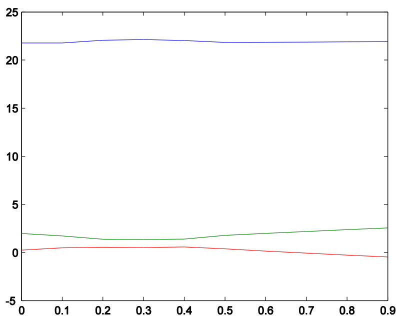
-Conjecture 1: If A is Lyapunov-stable, then A is D-stable.

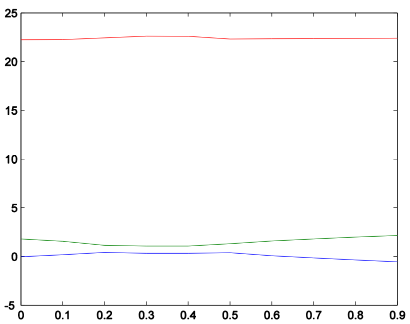
- Conjecture 2: If A is Lyapunov-stable, then any achievable equilibrium is globally stable.

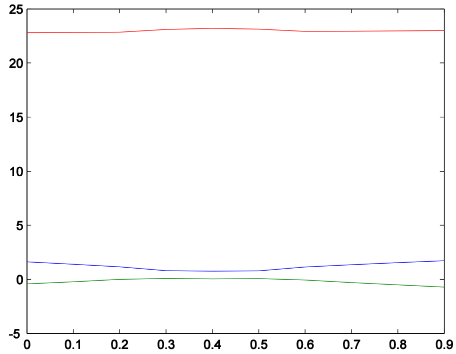
If the matrix A is positive definite, then it has already been shown in 'Ref [32,29]' that A is also D-stable. This proves conjecture (1). From 'Ref [33]', there exists a Lyapunov function for any achievable equilibrium point. This proves the second conjecture (2). Now, to check the stability of Lyapunov we carry out numerical simulations. To verify conjecture (1), we generated 500 samples of strictly positive *D* diagonal matrices and tested whether *DA* is still Lyapunov-stable.

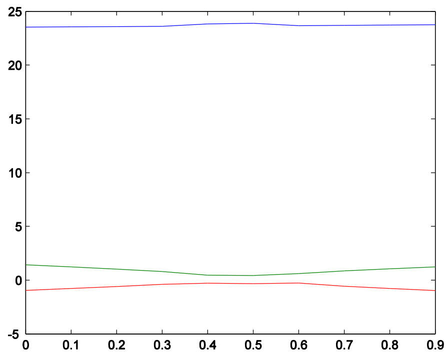


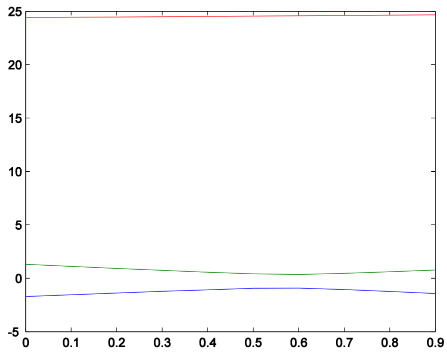


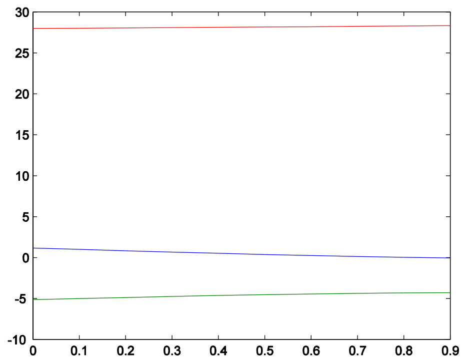












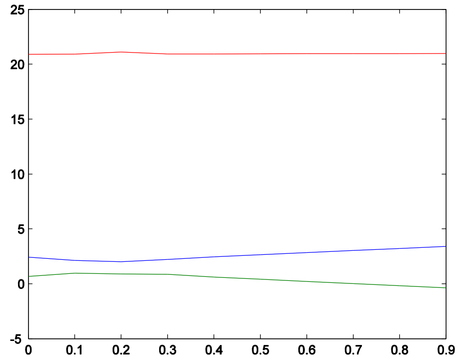


Figure 2. In this figure, we take the X-abscissa ρ_1 which belongs to the interval [0,0.9], to define the eigenvalues, fixing all the other parameters($\varepsilon_{11} = \varepsilon_{12} = 1; \varepsilon_{21} = 0$), and we vary the trade-off in each graph starting the reading from the left towards the right ; *l* is respectively equal to {0:9; 0:8; 0:7; 0:6; 0:5; 0:4; 0:3; 0; 1} as we take also ($S_1^* = 5$, $S_2^* = 3$, $C_1^* = 40$)

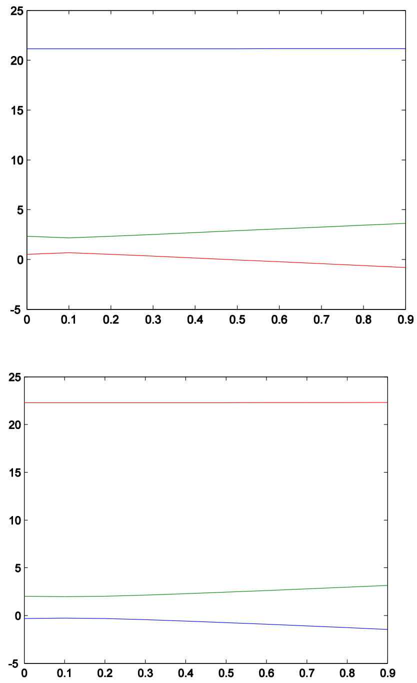
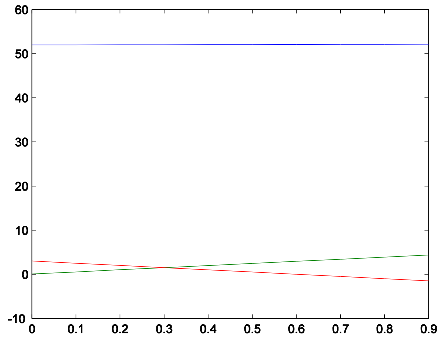


Figure 3. In this figure we take the same X-abscissa p1 and same value as figure one only we changed ε12 = 0; ε21 = 0 and l = 0.9 in the left graph; l = 0.6 in the right graph.



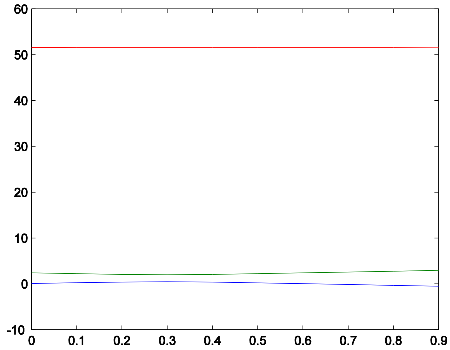


Figure 4. In this figure we have the same X-abscissa ρ1 and fixed l = 0.9 and ε12 = ε21 = ε11 = 1 and $S_1^* = 5$, $S_2^* = 5$, $C_1^* = 100$

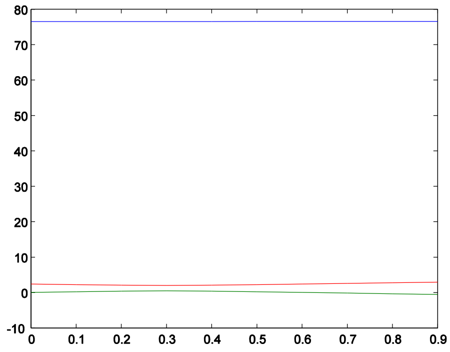


Figure 5. Here also we take the abscissa ρ_1, and $l = 0.9$, $\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = 1$ and $S_1^* = 3$; $S_2^* = 5$; $C_1^* = 150$.

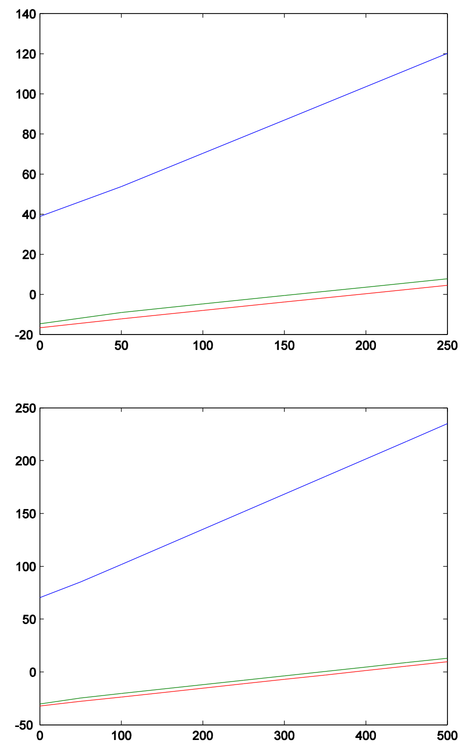


Figure 6. Here we change the abscissa from ρ_1, in the left graph we have S_1^* where $S_1^* = [0;250]$, in second graph we have S_1^* where $S_1^* = [0;500]$, we fixed $\rho 1 = 0.5$, and $l = 0.9$, $\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = 1$ and ,

$S_2^* = 5$; $C_1^* = 150$.

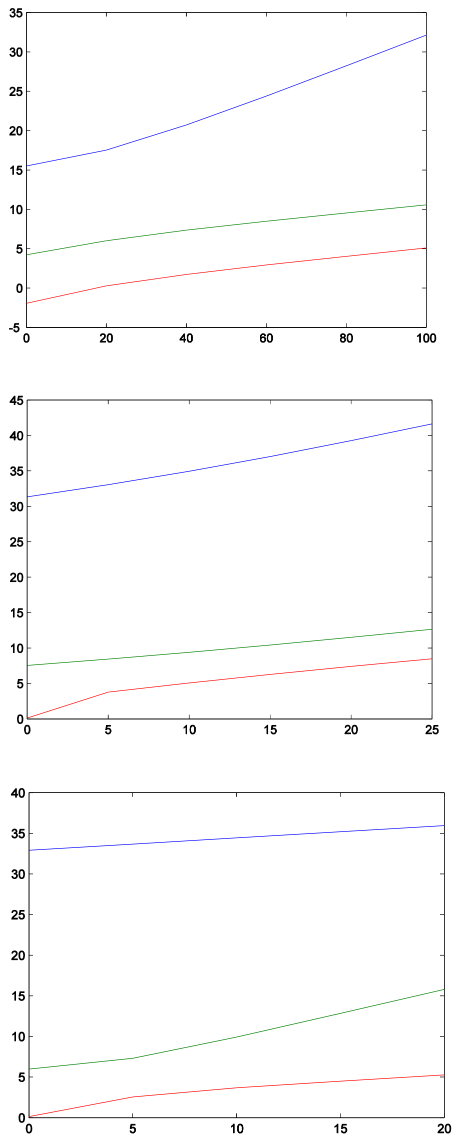
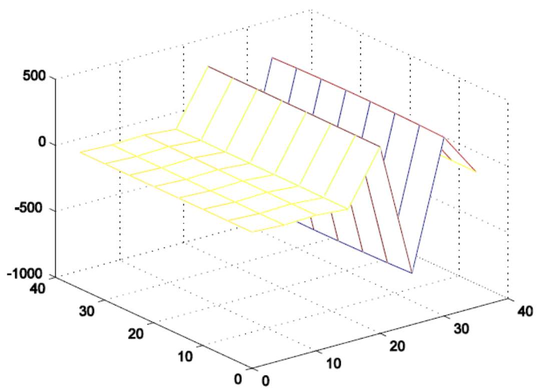


Figure 7. The abscissa for the first graph form the left is C_1^* where $C_1^* \in [0, 200]$ and $S_1^* = 10$; $S_2^* = 10$, and the abscissa for the second graph is $S_1^* \in [0; 25]$ and $S_2^* = 10$, $C_1^* = 100$ and the X-abscissa for the last graph is $S_2^* \in [0; 20]$ where $S_1^* = 10$, $C_1^* = 100$ and for the three graphs we fixe $\rho_1 = 0.2$, $l = 0.9$, $\varepsilon_{11} = 1$ and $\varepsilon_{21} = \varepsilon_{12} = 0$



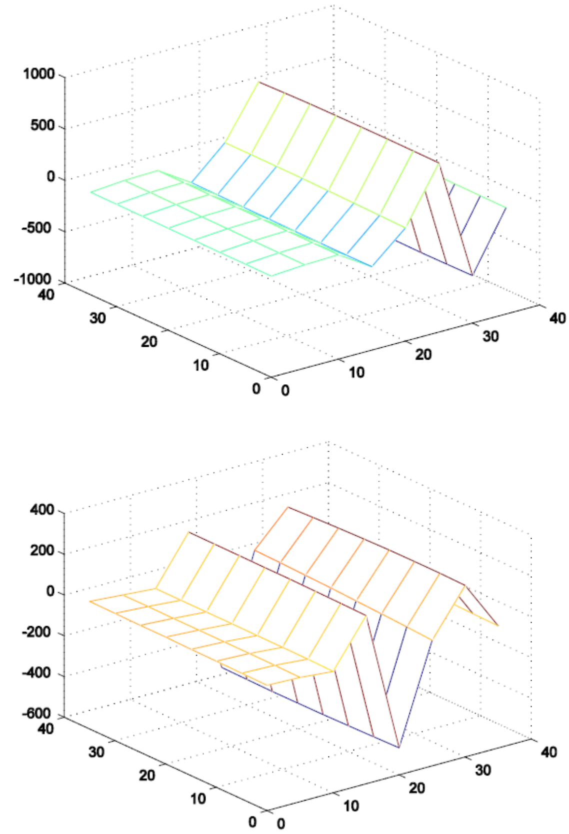
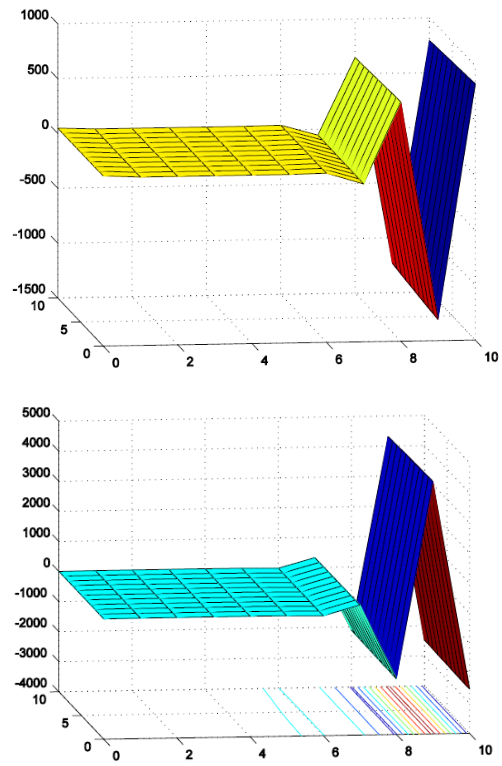


Figure 8. The first graph form the left define the result of the first Eigen-value; the second graph defines the results of the second Eigen-value, the third graph is the results third Eigen-value, all these three graphs are 3D curve withe the variables S_1^* and S_2^* where $S_1^* \in [0; 20]$; $S_2^* \in [0; 25]$ we fixed $C_1^* = 100$ and $\varepsilon_{11} = 1$ $\varepsilon_{21} = \varepsilon_{12} = 0$; $l = 0.9$; $\rho_1 = 0.2$.



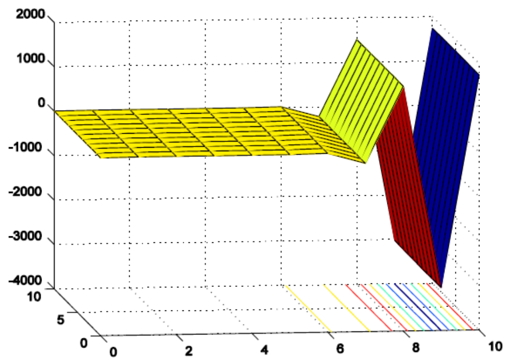


Figure 9. The first graph form the left defines the results of the first Eigen-value; the second graph defines the results of the second Eigen-value, the third graph is the results third Eigen-value, all these three graphs are 3D curve withe the variables S_1^* and S_2^* where $S_1^* \in [0;10]$; $S_2^* \in [0;10]$ we fixed $C_1^* = 100$ and $\varepsilon_{11} = 1$ $\varepsilon_{21} = \varepsilon_{12} = 0$; l = 0.9; $\rho_1 = 0.2$.

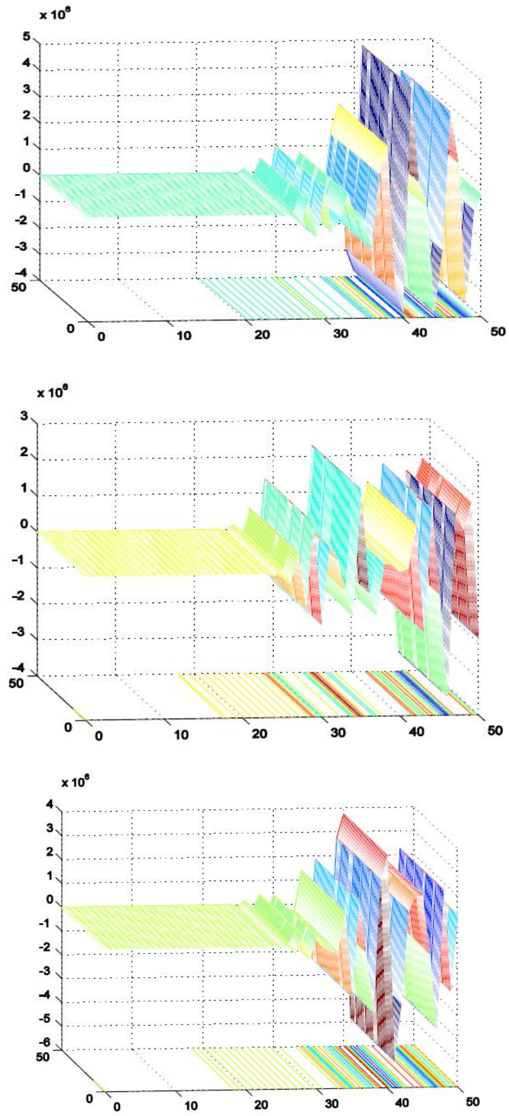


Figure 10. The first graph form the left defines the results of the first Eigen-value; the second graph defines the results of the second Eigen-value, the third graph is the results third Eigen-value, all these three graphs are 3D curve withe the variables S_1^* and S_2^* where $S_1^* \in [0;50]$; $S_2^* \in [0;50]$ we fixed $C_1^* = 500$ and $\varepsilon_{11} = 1$ $\varepsilon_{21} = \varepsilon_{12} = 0$; l = 0.9; $\rho_1 = 0.2$.

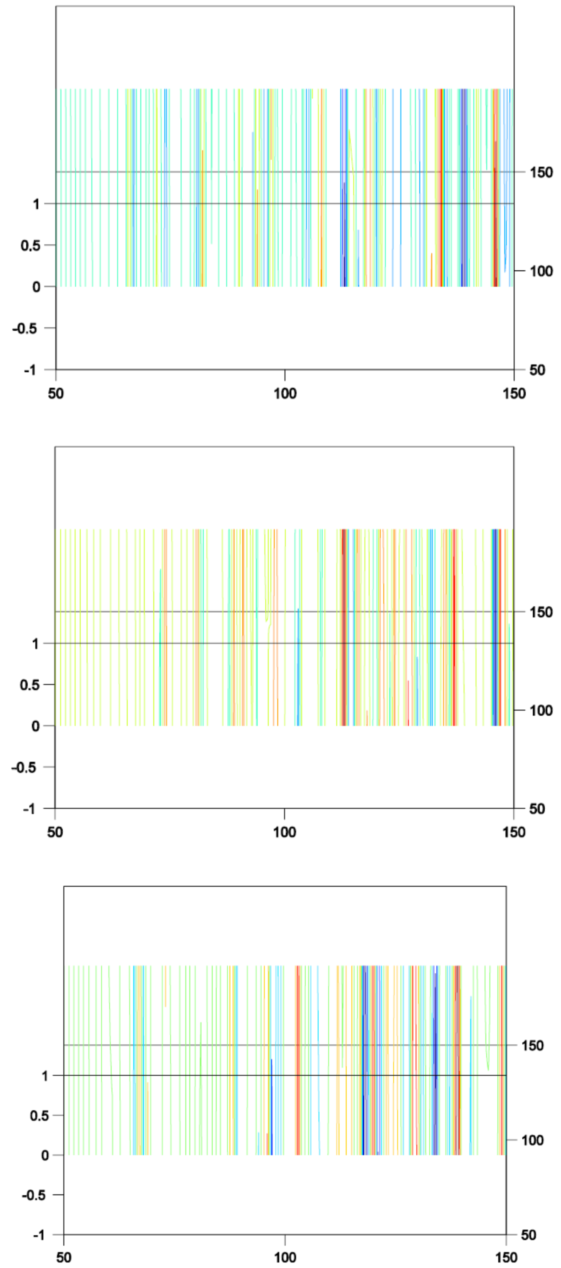


Figure 11. The first graph form the left defines the results of the first Eigen-value; the second graph defines the results of the second Eigen-value, the third graph is the results third Eigen-value, all these three graphs are 3D curve withe the variables S_1^* and S_2^* where $S_1^* \in [50;150]$; $S_2^* \in [50;150]$ we fixed $C_1^* = 1000$ and $\varepsilon_{11} = 1$ $\varepsilon_{21} = \varepsilon_{12} = 0$; l = 0.9; $\rho_1 = 0.2$.

III. Discussion

Complex dynamical systems in every field change from stability to the instability, here we focus on platform ecosystem system. When a platform continues growing, many perturbations can occur inside them, which drift toward the instability (situation we lose control on our system). To understand the mechanism of these changes we searched about the factors that mostly affect stability. To accomplish this goal a viable theory (Lyapunov stability theory) is used after reducing the model (to get rid from complexity like in [10, 18]). We focus in this paper on the linear case of our system i.e. $h = 0$.

This study analyzes whether the model's variables affect positively or negatively the stability state. Previous results show that: strong interaction (between suppliers and consumers), very big number of consumers comparing to number of suppliers, and the trade-off approaching to one are all helpful to attend the stabilization; Strong interspecific competition cause many perturbations inside the platform for example if there will be a sharp competition between a car's company with a phone's company will create a noise for the both companies. Some consumers buy their needs from supplier *i* some others buy from seller *j* the rest don't use platforms in their daily life because of the sharp interspecific competition suppliers may lose many consumers in a very small period, otherwise weak B_{ij} ; $_{j \neq i}$ help the system to be stable. As we mentioned strong interaction between consumers and suppliers is a positive factor in this situation there is a big demande from consumers, so the platform needs more suppliers to satisfy all consumers which helps the growth of platform and be more stable when suppliers can satisfy the demand, and to success on it, many suppliers prefer to lose a quality (what we name trade-off) in the aim to gain more consumers and be able to satisfy the demand in a short time for example reducing the cost is very attracting for consumers, suppliers in this situation have to reduce the quality of product in aim to gain more consumers and that makes them selling their product in a short time. All these are helpful factors for the stability platfrom and its safety. After using Lyapunov-stability theory (which we sited in the conjectures) we tested many times the stability, and show the important results in figures (2 to 11). The interval where the eigenvalues are all positive expands; in figures 3, 4 and 5 we base ourselves on changing the values of D and the ε_{ij} , it shows the importance of trade-off for stability and that strong competition in different commodities (or intraspecific competition) leads to the instability: for example a sharp competition between a car's company and phones' company will creates many perturbations that will affect negatively the stability of each company. Also interactions between consumers and suppliers are very important for stability. It is quite remarkable that if the number of consumers is largely greater than the number of suppliers our system tends to be stable faster; in figures 6 and 7 we change the X-axes from ρ_1 to S_1^* or S_2^* , C_1^* and these figures confirm more broadly what we observed in the previous figures; and to get closer to the best stability interval we built the curves in 3D we vary two factors at the same time in figures 8, 9, 10, and 11, and we notice that if the number of consumers is largely greater than the number of supplier and it approaches towards 1 ; ρ_1 approaches 0 and as long as there is a lot of interaction our system will stabilize easily.

IV. Diversity

Diversity is particularly important for collaboration, it helps provide better insight into the needs and motivations of all consumers, and it leads to more interaction inside and outside the platforms. This calculations following the logic used in 'Ref [19, 28]' and converting them to get those equation for the case of suppliers and consumers as follow:

$$a = M_1\ mean(B_{ij});\ a' = M_2\ mean(\lambda_{ij});$$
$$c^2 = M_1\ \mathrm{var}(B_{ij});\ c'^2 = M_2\ \mathrm{var}(\lambda_{ij});$$
$$d = corr(B_{ij}; B_{ji});\ d' = corr(\lambda_{ij}; \lambda_{ji})\ ;$$
$$\zeta^2 = \mathrm{var}(K_i);\ \zeta'^2 = \mathrm{var}(K_i').$$

Where *a* and *a'* are the antagonism competition and negative interaction within suppliers and within consumers respectively; *c* and *c'* are the heterogeneities; *d* and *d'* are the reciprocities and *ζ, ζ'* are the carrying capacities spread; Here we consider M_1 and M_2 as the large size of competition and negative interaction. d is the correlation between B_{ij} and B_{ji} , *d'* is the correlation between λ_{ij} and λ_{ji} , so that we have: $-1 \leq d \leq 1$ and $-1 \leq d \leq 1$. $d = 1$ and $d' = 1$ means that B_{ij} and λ_{ij} can have any distribution [19, 28].

When there is a mutualistic interaction within partners, it tends to increase the abundance; and then when the mutualistic partners have a high abundance, the beneficial effect of the interactions on the growth of partners would saturate.

We consider the following number of suppliers and consumers in the assembled state respectively as $M_1^* = M_1 * \phi; M_2^* = M_2 * \phi'$; in [28] they cited that coexistence decreases with increasing of heterogeneities and the carrying capacities spread, because more variances in carrying capacities and interactions implies that some suppliers are more likely to be more competitive overall than some other (and some consumers have more negative interaction within them overall than some others); also the reduction of reciprocities tends to increase the coexistence. As we also consider the Simpson Index $I_S = \left(\sum_i \frac{S_i}{S_{Tot}}\right)^2$ and $I_C = \left(\sum_i \frac{C_i}{C_{Tot}}\right)^2$ it gives a measure of the concentration of the biomass, its inverse, the diversity of Simpson indicates to us the efficiency of the diversity of the community of the biomass within all the suppliers and within all consumers.

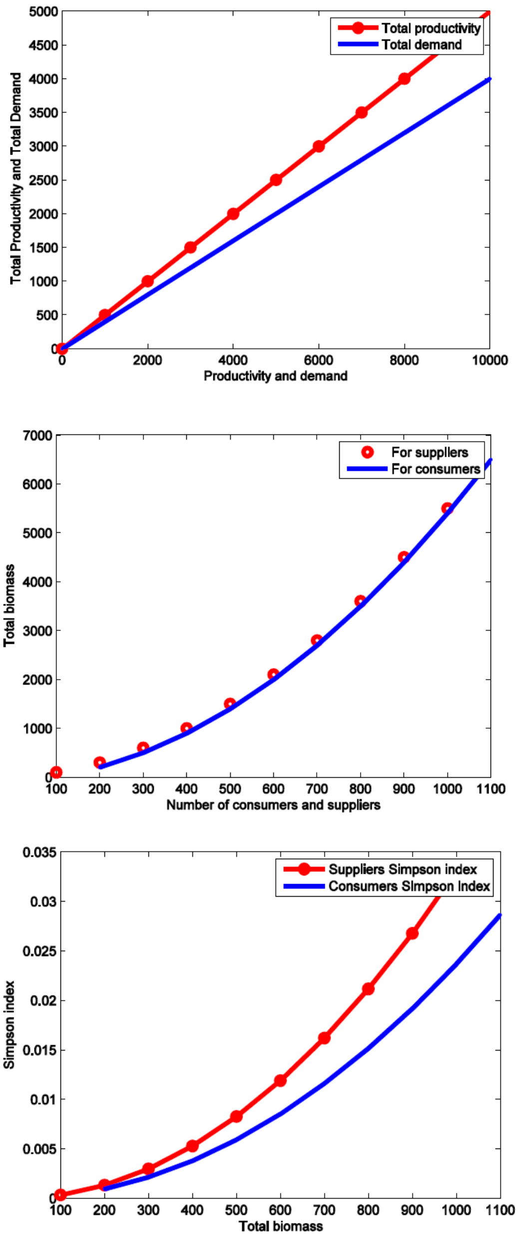


Figure 12. This figure shows at the left hand the total productivity and total demand assuming that $r_i = 0.5$ and μi = 0.6 and the graph in the right hand the total biomass for suppliers and consumers, while the last graph showing Simpson index.

V. The Monte Carlo Simulation

A. *The Inputs and Outcomes Parameters :*

Input parameters are the parameters that we pass to the service and are used during a service. Input parameters are used when dynamic content is passed to the external data source. Outcome parameters are the parameters that are fetched from the response of a service.

To get the confidence interval for :

$\langle B\rangle \in [B_{\min};\ B_{\max}]$; $\langle \delta\rangle \in [\delta_{\min};\ \delta_{\max}]$; and $\langle \lambda\rangle \in [\lambda_{\min};\ \lambda_{\max}]$ it is enough to get the confidence intervals for ρ_1 ; ρ_3 and ρ_2 respectively as:

$$\rho_1 \in \left[\overline{g_n} - Z(1)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}};\ \overline{g_n} + Z(1)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}}\right]$$

$$\rho_3 \in \left[\overline{(g_3)_n} - Z(3)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_3}}{\sqrt{n}};\ \overline{(g_3)_n} + Z(3)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_3}}{\sqrt{n}}\right]$$

$$\rho_2 \in \left[\overline{(g_2)_n} - Z(2)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_2}}{\sqrt{n}};\ \overline{(g_2)_n} + Z(2)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_2}}{\sqrt{n}}\right]$$

Now lets concentrate on the interactions between consumers and suppliers, first let M_{ij} the number of time that consumer *j* visiting supplier *i* in the platform; *M* can be regarded as a data matrix with m_s rows (number of suppliers) and mc column (number of consumers); this is the imputs of our calculation. We already mentioned the incidence matrix ε_{ij} where it equal to 1 if there is ineraction and 0 otherwise. M_{ij} is larger if $\varepsilon_{ij} = 1$ then if $\varepsilon_{ij} = 0$; and we expect there to be more consumers if the period of observation is longer, let *c* denote the large number of consumers that visiting suppliers in the platform; we write the number as it increase as $1 + a$ with $a > 0$, when $a = 0$ means no increase. The effect of overall time of observation by an overall constant *R* that multiplies the mean κ_{ij} (the mean number of observed consumer's *j* visiting suppliers *i* in the platform) and then:

$$\kappa_{ij} = R\,\nu_i\zeta_j\left(1 + a\,\varepsilon_{ij}\right) \tag{10}$$

The probability of observing exactly M_{ij} consumers, is drawn from a Poisson distribution with this mean is:

$$P\left(M_{ij}\middle|\kappa_{ij}\right) = \frac{\kappa_{ij}^{M_{ij}}}{M_{ij}!}e^{-\kappa_{ij}} \tag{11}$$

We have ν_i is the large number of suppliers *i*, and ζ_j is the large number of consumers *j*; (11) gives us the probability distribution of a single element M_{ij}, the we can combine (10) and (11) for all suppliers-consumers pairs to get the likelihood of the complete matrix *M*:

$$P\left(M\,\middle|\varepsilon,\theta\right) = \prod_{i,j}\frac{\left(R\,\nu_i\zeta_j\left(1 + a\,\varepsilon_{ij}\right)\right)^{M_{ij}}}{M_{ij}!}e^{-R\,\nu_i\zeta_j\left(1 + a\,\varepsilon_{ij}\right)} \tag{12}$$

The likelihood of (11) tells us the probability of *M* given ε and θ, now we want to know the probability of ε and θ given *M*, we apply the bay's rule in the form:

$$P\left(\varepsilon,\theta\,\middle|M\right) = \frac{P\left(M\middle|\varepsilon,\theta\right)P\left(\varepsilon\middle|\theta\right)P(\theta)}{P(M)} \tag{13}$$

For the prior on the probability on the network, $P\left(\varepsilon\middle|\theta\right)$. We make the conservative assumption in the absence of any knowledge to the contrary that all edges in the network are a priori just as likely. We denote the probability on an edge by *p*, and then the prior probability on the entire network is:

$$P\left(\varepsilon\middle|\theta\right) = \prod_{i,j}(1-p)^{1-\varepsilon_{ij}p^{\varepsilon_{ij}}} \tag{14}$$

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Let p is an additional parameter in the set of parameters (θ), then we assume a prior probability on p and assuming a uniform distribution, so the prior is constant and we can ignore it, then we'll be able to compute the prior probability (12); by considering the complete set of plausible structures, we can not only make an estimate of the network structure but else it can says how confident we are in that estimate, in effect putting error bars on the network. The probability that there is interaction between consumers j and suppliers i (or there is an edge) can be written as:

$$P\left(\varepsilon_{ij}=1\middle|M\right)=\sum_{\varepsilon}\int \varepsilon_{ij}P\left(\varepsilon,\theta\middle|M\right)d\theta \qquad (15)$$

This sum run over all possible incidence matrix ε_{ij} while the integral run over all parameters values, so the average of the function f of the matrix ε and set (θ) is:

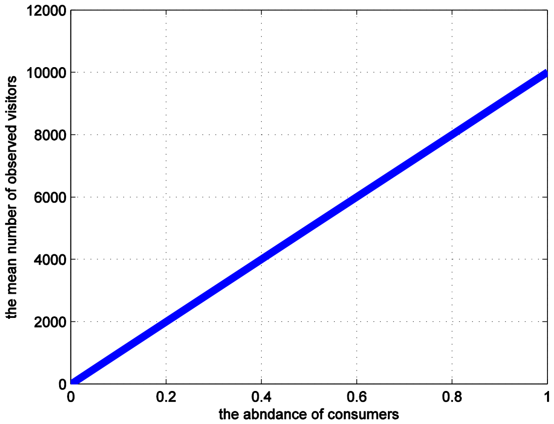
$$\left\langle f(\varepsilon,\theta)\right\rangle=\sum_{\varepsilon}\int f(\varepsilon,\theta)P\left(\varepsilon,\theta\middle|M\right)d\theta \qquad (16)$$

the matrix of incidence ε tells us about the structure of the network, but also θ reveal other important information that's why we consider it in (11), (12), (13) (14) and (15).

Eq.15 is not an easy task, so we use the Monte Carlo sampling technique to approximate it; we generate a sample of $\varepsilon|\ \theta$ pairs like $(\varepsilon_1,\theta_1),.....,(\varepsilon_n,\theta_n)$, where each pair appears with probability proportional to the posterior distribution j of (15), then we approximate the average of $f(\varepsilon,\theta)$:

$$\left\langle f(\varepsilon,\theta)\right\rangle \approx \frac{1}{n}\sum_{i=1}^{n} f(\varepsilon_i,\theta_i) \qquad (17)$$

Under many conditions, (16) will converge to its true value asymptotically as the number of Monte Carlo samples n becomes large; given ε and θ in (12) we can compute the likelihood $P\left(M\middle|\varepsilon,\theta\right)$ of M and then n samples possible data sets from these probability distribution.



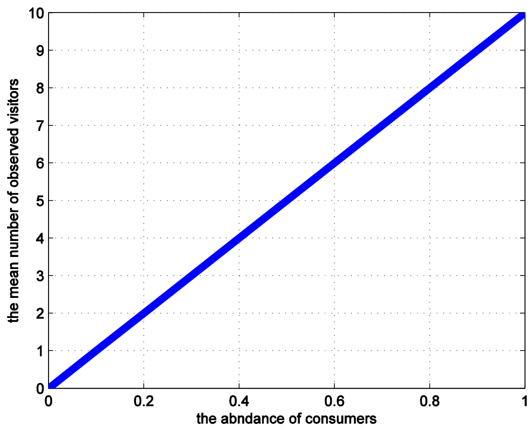


Figure 13. These two figures shows us the big difference between the mean number of observed visitors to the platform is the case of being interaction between consumers and suppliers and in the case of no interaction between them.

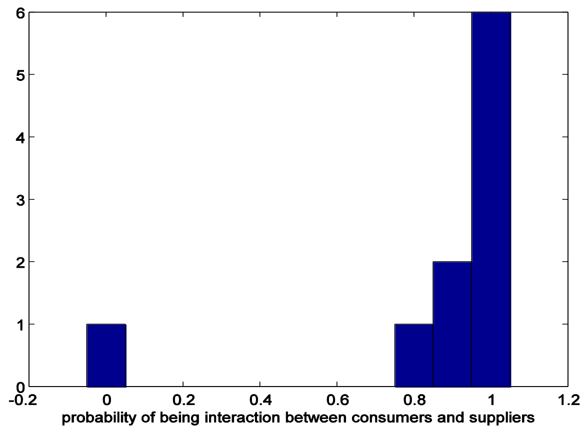


Figure 14. This figure show us that there is a high probability (approaching to 1) of being interaction between consumers and suppliers if consumers visit the platform.

VI. Conclusion

The aim of this paper is to develop an evolutional model for platform ecosystem through the time then reduce our system to get an effective model with effective number of consumers and effective number of suppliers which is the equilibrium of our reduced system any platform face with collapsing problems so trying to avoid it we test the stability of our system (stability is a safety factor) by using lyaponov theory and find out the condition that lead to the stability and what are the important factors in our system, then we took the help from Monte Carlo method to simulate the random variables and get their confidence or safety interval. First we build up the 2 differential equations referring to the variation of the number of suppliers and consumers within time then reduce it to get the effective model. The difficulties in this study are stability conditions we have to test it hundreds times to get good intervals; as Monte Carlo method is difficult because of the distributions, it is not easy to find the distribution low for random variables. Finally after finding stability conditions and simulating

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we can find a way to avoid collapsing which is the biggest problem that platform are facing with.

VII. Future Study

For a best future study based on this paper it is possible to test the stability in the non-linear case and with a real data.

Appendix: Supporting Information for "Stability and Its Determinants of Platform Ecosystem"

A. Introduction

a. Platfroms

There are several types of platforms such as Amazon, EBay, Google, Alibaba, Aliexpress, Taobao,...etc; The notion of platform has been developed by management researchers in three different domain of research (product, technological system, and transactions) [6]. Platforms are often associated with the "network Effects": that is, the more users adopt the platform, the more valuable the platform becomes to the owner and users due to increasing access to the network of users and often to a growing set of complementary innovations In other words, there are more and more incentives to more companies and users to adopt the platform and join Ecosystem with the arrival of more users and add-ons [22].

B. Derivation of the 2D Reduced Model

First of all from (4), we can obtain the effective average number of suppliers and consumers and we can write:

$$r_i S_i \approx r\, S_{eff} \qquad (18)$$

And

$$\mu_i C_i \approx \mu\, C_{eff} \qquad (19)$$

Here we define S_{eff} and C_{eff} as the effective number of suppliers and consumers (respectively). Suppliers in different platforms and different commodities do not compete as those in same platfrom and same commodities, for that we can write: $B_{ii} >> B_{ij}$; also we can generate it for those who interact positively within them if they are in same commodities or different, so the positive interaction in same commodities will be stronger than that on in different commodities, and same situation for consumers, then we can write: $\delta_{ii} >> \delta_{ij}$ and $\lambda_{ii} >> \lambda_{ij}$. In other side we can write:

$$\sum_{j=1}^{M_1} B_{ij} S_j S_i \approx B\, S_{eff}^2 \qquad (20)$$

And

$$\sum_{j=1}^{M_1} \delta_{ij} S_j S_i \approx \delta\, S_{eff}^2 \qquad (21)$$

And

$$\sum_{j=1}^{M_2} \lambda_{ij} C_j C_i \approx \lambda\, C_{eff}^2 \qquad (22)$$

To integrate interspecific interaction in our model, we write the interactions terms as follow:

$$\sum_{j=1}^{M_1} B_{ij} S_j S_i \approx \frac{\sum_{i=1}^{M_1}\sum_{i=1}^{M_1} B_{ij}}{\sum_{i=1}^{M_1} 1} S_{eff}^2 \approx B\, S_{eff}^2 \qquad (23)$$

And

$$\sum_{j=1}^{M_1} \delta_{ij} S_j S_i \approx \frac{\sum_{i=1}^{M_1}\sum_{i=1}^{M_1} \delta_{ij}}{\sum_{i=1}^{M_1} 1} S_{eff}^2 \approx \delta\, S_{eff}^2 \qquad (24)$$

And

$$\sum_{j=1}^{M_2} \lambda_{ij} C_j C_i \approx \frac{\sum_{i=1}^{M_2}\sum_{i=1}^{M_2} \lambda_{ij}}{\sum_{i=1}^{M_2} 1} C_{eff}^2 \approx \lambda\, C_{eff}^2 \qquad (25)$$

Now for finding the effective interaction in the network of our model in both sides (suppliers side and consumers side), we start by calculating the strength of the mutualistic interaction for each group of suppliers and consumers as follows:

$$\sum_{j=1}^{M_2} \gamma_{ij}^{S_i} C_j \approx \sum_{j=1}^{M_2} \frac{\gamma_0}{(G_i)^l} \varepsilon_{ij} C_j \approx \gamma_0 G_i^{1-l} C_{eff} \qquad (26)$$

And

$$\sum_{j=1}^{M_1} \gamma_{ij}^{C_i} S_j \approx \sum_{j=1}^{M_1} \frac{\gamma_0}{(Z_i)^l} \varepsilon_{ij} S_j \approx \gamma_0 Z_i^{1-l} S_{eff} \qquad (27)$$

There are many ways and methods to get the average of the mutualistic strength, in this work we use the unweighted method and we find:

$$\left\langle \gamma_{ij}^{S_i} \right\rangle = \frac{\sum_{i=1}^{M_2} \gamma_0 G_i^{1-l}}{\sum_{i=1}^{M_2} 1} \qquad (28)$$

And

$$\left\langle \gamma_{ij}^{C_i} \right\rangle = \frac{\sum_{i=1}^{M_1} \gamma_0 Z_i^{1-l}}{\sum_{i=1}^{M_1} 1} \qquad (29)$$

C. The Steady State Solution of Suppliers-Consumers

To obtain the equilibrium point (steady state) solution of suppliers-consumers number from our reduced model is by solving two equations which are: $\frac{dS_{eff}}{dt} = 0$, and $\frac{dC_{eff}}{dt} = 0$ we have:

$$\frac{dS_{eff}}{dt} = rS_{eff} - BS_{eff}^2 + \delta S_{eff}^2 + \frac{\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}}{1 + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}} S_{eff} = 0 \qquad (30)$$

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$$\frac{dC_{eff}}{dt} = \mu C_{eff} - \lambda C_{eff}^2 + \frac{\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}}{1 + h\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}} C_{eff} = 0 \qquad (31)$$

As we can define the Jacobian matrix related to the equilibrium point solution in the way:

$$J = \begin{bmatrix} \frac{df}{dS_{eff}} & \frac{df}{dC_{eff}} \\ \frac{dg}{dS_{eff}} & \frac{dg}{dC_{eff}} \end{bmatrix} \qquad (32)$$

$$J = \begin{bmatrix} 2S_{eff}(\delta - B) + r + \frac{\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}}{1 + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}} & \frac{\left\langle \gamma_{ij}^{(S_i)} \right\rangle \left(\left[S_{eff} + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle S_{eff} C_{eff} \right] - C_{eff} h \left\langle \gamma_{ij}^{(S_i)} \right\rangle \right)}{\left(1 + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}\right)^2} \\ \frac{\left\langle \gamma_{ij}^{(C_i)} \right\rangle \left(\left[C_{eff} + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff} S_{eff} \right] - S_{eff} h \left\langle \gamma_{ij}^{(C_i)} \right\rangle \right)}{\left(1 + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}\right)^2} & -2\lambda C_{eff} + \mu + \frac{\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}}{1 + h\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}} \end{bmatrix} \qquad (33)$$

After solving eq.30 and eq.31 we get those result:

$$S_{eff} = \frac{-\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff} - r\left[1 + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}\right]}{\left[1 + h\left\langle \gamma_{ij}^{(S_i)} \right\rangle C_{eff}\right](\delta - B)} \qquad (34)$$

$$S_{eff} = \frac{-\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff} - \mu\left[1 + h\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}\right]}{\left[1 + h\left\langle \gamma_{ij}^{(C_i)} \right\rangle S_{eff}\right](-\lambda)} \qquad (35)$$

D. Stability

In this work we discuss the numerical results obtained after doing numerical experiments on the conditions of stability in the linear case

$$\begin{bmatrix} \frac{dS}{dt} \\ \frac{dC}{dt} \end{bmatrix} = Diag\left(\begin{bmatrix} S \\ C \end{bmatrix} \right) \times \left(\begin{bmatrix} r \\ \mu \end{bmatrix} - \begin{bmatrix} B - \delta & -\gamma^S \\ -\gamma^C & \lambda \end{bmatrix} \right)$$

The interaction strength matrix A is in the form:

$$A = \begin{bmatrix} B - \delta & -\gamma^S \\ -\gamma^C & \lambda \end{bmatrix}$$

In figure 1 we take on the x-axis S_{eff} and C_{eff} and we fix the other variables, the results of figure 1 show that our equilibrium is well positive which makes it feasible, in this study it is very important to find an achievable balance because it directly influences the stability of our system and this is what we will see in the next figure; the goal of figures 2 to 11 is to approach the best values of our factors which fall under the conditions of stability (conjecture 1) of our system, in all these figures we reduce the dimensions of the matrices to facilitate the obtaining desired results. In Figure 2 we take ρ_1 in the X-axis and in the Y-axis the eigenvalues, and we fixe all the other factors,

$$B = \begin{bmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{bmatrix}; \delta = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}; \lambda = [0.5]; \gamma^s = \begin{bmatrix} \frac{\varepsilon_{11}}{G_1^{\wedge l}} \\ \frac{\varepsilon_{21}}{G_2^{\wedge l}} \end{bmatrix}; \gamma^c = \begin{bmatrix} \frac{\varepsilon_{11}}{Z_1^{\wedge l}} & \frac{\varepsilon_{12}}{Z_1^{\wedge l}} \end{bmatrix}$$

And $D = \begin{bmatrix} S_1^* & 0 & 0 \\ 0 & S_2^* & 0 \\ 0 & 0 & C_1^* \end{bmatrix}$ we have in figure 2 it is easy to notice that when l approaches to 1 and ρ_1 approaches 0.

In this part we have *a* and *a'* are the antagonism competition and negative interaction within suppliers and within consumers respectively; *c* and *c'* are the heterogeneities; *d* and *d'* are the reciprocities and ζ, ζ' are the carrying capacities spread. Because of the absence of the empirical information about the competition within suppliers and negative interaction within consumers, we use a mean field approximation for the competition and negative interaction parameter respectively. $B_{ij} = b_{ij}\frac{r_i}{K_i}$; $\lambda_{ij} = b_{ij}^{'}\frac{\mu_i}{K_i^{'}}$.

We have the total Biomass for suppliers and consumers respectively equal to: $T_B = \sum_i S_i$; $T_B^{'} = \sum_i C_i$.

The total productivity and total demand are respectively equal to: $T_P = \sum_i r_i S_i$; $T_P^{'} = \sum_i \mu_i C_i$.

The Simpson index equal to: $D^{-1} = \sum_i \left(\frac{S_i}{T_B}\right)^2$; $D'^{-1} = \sum_i \left(\frac{C_i}{T_B^{'}}\right)^2$.

The variability equal to: $V = \sum_i \left(\frac{\text{var}(S_i)}{M_1}\right)^2$; $V' = \sum_i \left(\frac{\text{var}(C_i)}{M_2}\right)^2$.

E. Monte Carlo

We assume previously $B_{ii} = 1$ and B_{ij} = _1 if $i \neq j$ so we focus on ρ_1, so for getting the confidence interval for $\langle B \rangle$ is enough to get ρ_1's confidence interval $\rho_1 \in [0,\ 1[$; it is easy to see that ρ_1 follows the uniform distribution, the density function of ρ_1 is $f\left(\rho_{1i}^{'}\right)_{i=\overline{1,N}}$, is defined as $f\left(\rho_{1i}^{'}\right) = 1$ if $\rho_1 \in [0,\ 1[$ and

$f\left(\rho_{1i}^{'}\right)=0$ otherwise. His math expectation is: $\mathrm{E}\left(f\left(\rho_{1i}^{'}\right)\right)=\frac{1}{2}$ and his variance equal to: $\frac{1}{12}$. Now if we have n-samples of ρ_1 which is $\left(\rho_{1,...,}^{'}\rho_n^{'}\right)$.

We have $f_\rho\left(\rho_{1i}^{'}\right)$ is greater than 0, on a set of values X, the expected value of a function g of ρ_1 is:

$$G=\mathrm{E}\left(g\left(\rho_1\right)\right)=\int_{\rho_{1i}^{'}\in X} g\left(\rho_{1i}^{'}\right) f\left(\rho_{1i}^{'}\right) d\rho_{1i}^{'} \qquad (36)$$

Now we take n-sample of ρ_1 and we compute the mean of $g\left(\rho_{1i}^{'}\right)$, so the Monte Carlo estimates:

$$\left(\bar{g}_n\left(\rho_{1i}^{'}\right)=\frac{1}{n}\sum_{i=1}^{n} g\left(\rho_{1i}^{'}\right)\right) \qquad (37)$$

of $\mathrm{E}\left(g\left(\rho_1\right)\right)$, as we have a random variable

$$\left(\bar{g}_n\left(\rho_1\right)=\frac{1}{n}\sum_{i=1}^{n} g\left(\rho_1\right)\right) \qquad (38)$$

Which we call the Monte Carlo estimator of $g\left(\rho_{1i}^{'}\right)$

There exists a weak law of a large numbers, for any arbitrary small ε that:

$$\lim_{n\to+\infty} P\left(\left|\bar{g}_n\left(\rho_1\right)-\mathrm{E}\left(g\left(\rho_1\right)\geq\right)\varepsilon\right|\right)=0 \qquad (39)$$

'Ref [34]'Tells us that as n gets large, than there is a small probability $\bar{g}_n\left(\rho_1\right)$ deviates from $\mathrm{E}\left(g_n\left(\rho\ \right)\right)$, also we summarize in 'Ref [34]' that the law of a large numbers says that as long as n is large enough, $\bar{g}\ (\rho')$ resulting from a riding from a Monte Carlo experiment must be close to $\mathrm{E}\left(g\left(\rho_1\right)\right)$; we have $\mathrm{E}\left(\bar{g}_n\right)=G=\mathrm{E}\left(g\ \left(\rho_1\right)\right)$ and at this point $\bar{g}_n\left(\rho\right)$ is unbiased for $\mathrm{E}\left(g\left(\rho_1\right)\right)$.

The precision of this estimation via the variance of $\bar{g}_n$, we assume that our sample is iid, then this variance is estimated using the empirical variance:

$$S^2_{g(\rho)}=\frac{1}{n}\sum_{i=1}^{n}\left(g\left(\rho_{1i}^{'}\right)-\bar{g}_n\right)^2=\sigma_g^2 m \qquad (40)$$

With

$$S_g^2=\mathrm{E}\left(g^2\left(\rho_1\right)\right)-\left(g\ \left(\rho_1\right)\right)^2=\int_X g^2\left(\rho'\right) f_{\rho_1}\left(\rho'\right) d\rho_1^{'}-G^2 \qquad (41)$$

by the central limit theorem, we know that the variable $Z:=\frac{\bar{g}_n-G}{\sigma_g/\sqrt{n}}\sim N\left(0,1\right)$, therefore it is possible to build a confidence interval, which allows to frame the error made by replacing G by $\bar{g}_n$, we denote the error

$\in_n$, for a given level of risk α then we have $\left|\in_n\right|\leq Z_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}}$, this method therefore makes it possible to quantify the error made on condition of estimating σ_g by its empirical counterpart $\hat{\sigma}_g=\sqrt{S^2_{g(\rho_1)}}$

The number of simulations n necessary to reach a desired margin of error which is: $\left|\in_n\right|\leq Z\left(1\right)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}}$ whenever $\in_n$ is smaller, n is very large.

We have :

$$P\left(\bar{g}_n-Z\left(1\right)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}}\leq G\leq\bar{g}_n+Z\left(1\right)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}}\right)=1-\alpha \qquad (42)$$

So the confidence interval is:

$$I_C=\left[\bar{g}_n-Z\left(1\right)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}};\ \bar{g}_n+Z\left(1\right)_{1-\frac{\alpha}{2}}\frac{\sigma_g}{\sqrt{n}}\right] \qquad (43)$$

Because of missing information δ about λ and we suppose that the mean of δ equal to $\left\langle\delta_{ii}\right\rangle=\rho_3$ and $\left\langle\delta_{ij}\right\rangle=a_2$, and the mean of λ equal to: $\left\langle\lambda_{ii}\right\rangle=\rho_2$ and $\left\langle\lambda_{ij}\right\rangle=a_1$ with: $\rho_3\in\left[0\quad\rho_1\right[$ and $\rho_2\in\left[0\quad 1\right[$. As we did previously ρ_3 and ρ_2 follow the uniform distribution on $\left[0\quad\rho_1\right[$ and [0 ; 1[, with the Monte Carlo simulation we find: first we have the expectation of each of ρ_3 and ρ_2 equal to $\frac{\rho_1}{2}$ and $\frac{1}{2}$ respectively and their variance equal to $\frac{\left(\rho_1\right)^2}{12}$ and $\frac{1}{12}$ respectively for getting their I_c which are equal to:

$$\rho_3\in\left[\overline{(g_3)_n}-Z\left(3\right)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_3}}{\sqrt{n}};\ \overline{(g_3)_n}+Z\left(3\right)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_3}}{\sqrt{n}}\right]$$

$$\rho_2\in\left[\overline{(g_2)_n}-Z\left(2\right)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_2}}{\sqrt{n}};\ \overline{(g_2)_n}+Z\left(2\right)_{1-\frac{\alpha}{2}}\frac{\sigma_{g_2}}{\sqrt{n}}\right]$$

The probability of the network having incidence matrix ε given by matrix M is:

$$P\left(\varepsilon,\theta\left|M\right.\right)=\frac{P\left(M\left|\varepsilon,\theta\right.\right)P\left(\varepsilon\left|\theta\right.\right)P\left(\theta\right)}{P\left(M\right)} \qquad (44)$$

Here θ are model parameters, $P\left(M\right)$ is an important normalizing constant. M_{ij} of the matrix M refers the number of times that consumers j visiting suppliers i in the platform, while $\varepsilon=0$ or 1 refers to the absence or presence of interactions between consumers and suppliers (respectively), M and ε are a M_1 * M_2 dimension matrices, where M_1 is the number of suppliers in the network while M_2 is the number of consumers in the network. We model the number of consumers visiting suppliers in the platform as a Poisson random variable with mean:

$$\kappa_{ij}=R\,\nu_i\zeta_j\left(1+a\ \varepsilon_{ij}\right) \qquad (45)$$

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Assuming Uniform priors on all the parameters and edges that are a priori equally likely with probability *p*, then we find:

$$P(\varepsilon,\theta|M) \propto \prod_{ij}(1-p)^{1-\varepsilon_{ij}} p^{\varepsilon_{ij}} \frac{\kappa_{ij}^{M_{ij}}}{M_{ij}!} e^{-\kappa_{ij}} \quad (46)$$

From eq.46 and paper [35] we could employ an expectation maximization algorithm to calculate the distribution over potential network structures and a point estimate.

The sample values of parameter _ from the marginal distribution:

$$P(\varepsilon,\theta|M) = \sum_{\varepsilon} P(\varepsilon,\theta|M) \quad (47)$$

Eq.46 can gives:

$$P(\theta|M) \propto e^{-R} \prod_{ij} (R\nu_i\zeta_i)^{M_{ij}} \left[1-p+p(1+a)^{M_{ij}} e^{-R\nu_i\zeta_i a}\right] \quad (48)$$

By using Hamiltonian Monte Carlo method we can simple this distribution, and it gives our estimates of the parameter values themselves; for given θ we can estimate the network by sampling from the distribution:

$$P(\varepsilon|M,\theta) = \frac{P(M|\varepsilon,\theta)P(\varepsilon|\theta)}{P(M|\theta)} \quad (49)$$

Using $P(M|\varepsilon,\theta)$ and $P(\varepsilon|\theta)$ and noting that $P(M|\theta)$ is proportional to eq.48, we get:

$$P(\varepsilon|M,\theta) = \frac{\prod_{ij}(1-p)^{1-\varepsilon_{ij}} + \left[p(1+a)^{M_{ij}} e^{-R\nu_i\zeta_j a}\right]^{\varepsilon_{ij}}}{\prod_{ij}(1-p) + \left[p(1+a)^{M_{ij}} e^{-R\nu_i\zeta_j a}\right]} = \prod_{ij} Q_{ij}^{\varepsilon_{ij}} (1-Q_{ij})^{1-\varepsilon_{ij}} \quad (50)$$

Where:

$$Q_{ij} = P(\varepsilon_{ij}=1|M,\theta) = \frac{p(1+a)^{M_{ij}} e^{-R\nu_i\zeta_j a}}{(1-p)+\left[p(1+a)^{M_{ij}} e^{-R\nu_i\zeta_j a}\right]} \quad (51)$$

Eq.51 is the posterior probability of $\varepsilon_{ij}=1$ given θ ; so we can simply average Q_{ij} over our θ to get the expected probability of being interaction between consumers and suppliers in the platform. More generally we can compute an estimate of any function $f(\varepsilon,\theta)$ by drawing *m* samples θ_k of θ and *n* random $\varepsilon_l(\theta_k)$; for each $\varepsilon_1,\ldots,\varepsilon_l$ are iid, with Q_{ij} given by eq.51, then the average is:

$$\langle f(\varepsilon,\theta)\rangle \approx \frac{1}{m.n} \sum_{k=1}^{m} \sum_{l=1}^{n} f(\varepsilon_l(\theta_k),\theta_k) \quad (52)$$

eq.48 has the form:

$$\log(\theta|M) = -R + \sum_{ij}(x_{ij}+y_{ij}) \quad (53)$$

Where:

$$x_{ij} = M_{ij} \log R\nu_i\zeta_j \quad (54)$$

And

$$y_{ij} = \log\left((1-p)+\left[p(1+a)^{M_{ij}} e^{-R\nu_i\zeta_j a}\right]\right) \quad (55)$$

To avoid a potential over or underflow and ensure numerical stability we rewrite the latter expression slightly by defining:

$$\kappa_{ij} = \log(1-p) \quad (56)$$

And

$$v_{ij} = \log(p) + M_{ij} \log(1+a) - R\nu_i\zeta_j a \quad (57)$$

Then we can write:

$$y_{ij} = \begin{cases} \kappa_{ij} + \log\left(1+e^{v_{ij}-\kappa_{ij}}\right) & if \quad \kappa_{ij} > v_{ij} \\ v_{ij} + \log\left(1+e^{\kappa_{ij}-v_{ij}}\right) & if \quad v_{ij} \geq \kappa_{ij} \end{cases} \quad (58)$$

Eq.57 ensures that y_{ij} is always a manageable number.

CONFLICT OF INTEREST

There is no conflict of interest.

AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: Doing methodology, analysis and writing the paper: LAMIA LOUDAHI. Correcting the English, paper's structure and give instructions about economic information's: ADNAN KHURSHID.

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